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MATHEMATICS

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SECTION-I
TOPIC WISE PROBLEMS

TOPIC 1

STRAIGHT LINE

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

1.1 If two curves whose equations are $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and $a'x^2 + 2h'xy + b'y^2 + 2g'x + 2f'y + c' = 0$ intersect in four concyclic points, then :-

(1) $\frac{a-a'}{h} = \frac{b-b'}{h'}$ (2) $\frac{a-b}{h} = \frac{a'-b'}{h'}$ (3) $\frac{a-b}{h'} = \frac{a'-b'}{h}$ (4) $\frac{a-b}{hgf} = \frac{a'-b'}{h'g'f'}$

1.2 If a straight line through the point P(3, 5) makes an angle $\frac{\pi}{6}$ with x-axis and meets the lines $2x + y + 5 = 0$ & $3x - 2y + 7 = 0$ at Q & R then $\frac{PQ}{PR} =$

(1) $\frac{3\sqrt{3}-2}{2\sqrt{3}+1}$ (2) $\frac{3\sqrt{3}+1}{3\sqrt{3}-2}$ (3) $\frac{2(12\sqrt{3}-8)}{6\sqrt{3}+3}$ (4) $\frac{6\sqrt{3}+3)}{(12\sqrt{3}-8)}$

1.3 The equations of two equal sides AB & AC of an isosceles ΔABC are $x + y = 5$ & $7x - y = 3$ respectively, the equation of the side BC if ar ΔABC is 5 units is

(1) $3x + y + 2 = 0$ (2) $3x + y + 12 = 0$ (3) $x - 3y + 1 = 0$ (4) $x + 3y + 20 = 0$

1.4 A triangle is formed by joining three points A(8, 2), B(–4, –4) and C(16, 1) on rectangular hyperbola $xy = 16$. If orthocentre of the triangle ABC is (h,k) then product hk is equal to

(1) 4 (2) 8 (3) 16 (4) 32

1.5 The triangle with vertex (1, 5), (– 3, 1) & (– 2, 1) is

(1) isosceles triangle (2) equilateral triangle (3) right angle triangle (4) None of these

1.6 In which ratio x-axis divides the line segment joining (1, 2) & (2, 3)

(1) 3 : 2 (2) 2 : 3 (3) – 3 : 2 (4) – 2 : 3

1.7 Harmonic conjugate of (0, 0) wrt to (– 1, 0) and (2, 0) is

(1) (4, 0) (2) (– 4, 0) (3) (3, 0) (4) (– 3, 0)

1.8 If $A(\cos\theta_1, \sin\theta_1)$, $B(\cos\theta_2, \sin\theta_2)$ & $C(\cos\theta_3, \sin\theta_3)$, then orthocenter of ΔABC is

(1) $\left(\frac{\Sigma\cos\theta_1}{2}, \frac{\Sigma\sin\theta_1}{2}\right)$ (2) $\left(\frac{\Sigma\cos\theta_1}{3}, \frac{\Sigma\sin\theta_1}{3}\right)$ (3) $(\Sigma\cos\theta_1, \Sigma\sin\theta_1)$ (4) Data insufficient

1.9 Equation of median through vertex B of ΔABC where A(0, 0), B(0, 1) & C(1, 0) is

(1) $y + 2x = 1$ (2) $2y + 2x = 1$ (3) $x + y = 1$ (4) $3x + 2y = 2$

1.10 Normal form of line $x + y + 1 = 0$ is

(1) $x\cos(45°) + y\sin(135°) + \frac{1}{\sqrt{2}} = 0$ (2) $x\cos(45°) + y\sin(45°) = \frac{1}{\sqrt{2}}$

(3) $x\cos(225°) + y\sin(225°) = \frac{1}{\sqrt{2}}$ (3) $x\cos(45°) + y\sin(45°) + \frac{1}{\sqrt{2}} = 0$

1.11 If in a ΔABC, B is the orthocenter and if circumcenter of ΔABC is (2, 4) and vertex A is (0, 0) then coordinate of vertex C is

(1) (4, 2) (2) (4, 8) (3) (8, 4) (4) (8, 2)

1.12 The equation of line passing through origin & at an angle of 30° with $y = \frac{1}{\sqrt{3}}x + 1$

(1) $y - \sqrt{3}\,x$ (2) $\sqrt{3}\,y = x$ (3) $-\sqrt{3}\,y = x$ (4) $y = \sqrt{3}\,x$

1.13 If $y = \sqrt{3}x + 1$ is a angle bisector of L_1 & L_2 & if L_1 is $y = \frac{x}{\sqrt{3}} + 1$ then equation of L_2 is

(1) x = 0 (2) y = 0 (3) x = 1 (4) y = 1

1.14 Area of square having two sides y = x + 1 and y = x + 2 is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 4

1.15 Area of polygon having sides
y – x – 1 = 0, 2y – 2x + 2 = 0, y – 2x –2 = 0 and 3y – 6x – 9 = 0

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 4 (4) 2

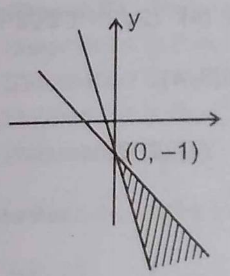
1.16 If area of ΔABC 5 sq. unit where A(1, 1), B(2,2) and C lies on y = 2x then co-ordinate of C can be

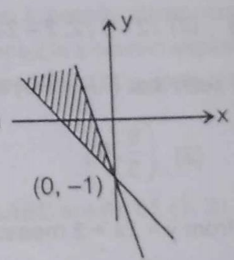
(1) (–10, –20) (2) (–10, 20) (3) (10, –20) (4) (20, 10)

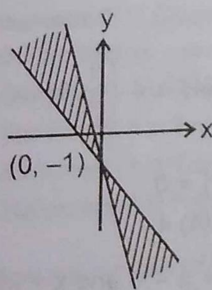
1.17 Area of pentagon formed by (1, 2), (2, –1), (–2, –1), (2, 1) and (– 1, 2)

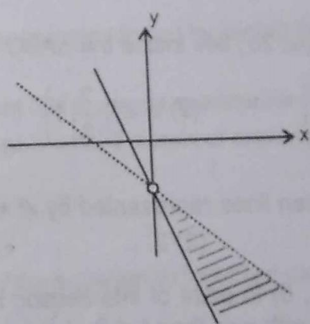
(1) 20 (2) 15 (3) 5 (4) 10

1.18 Area included by y + x + 1 < 0 and y + 2x +1 ≥ 0 is

(1) 

(2) 

(3) 

(4) 

1.19 Point on line x + y = 4 which is at a unit distance from the line 4x + 3y – 11 = 0 is

(1) (4, 0) (2) (0, 4) (3) (–6, 10) (4) (10, – 8)

1.20 Mirror image of (–3, 5) in line mirror $x - y + 2 = 0$
(1) (–1, 3) (2) (1, 3) (3) (3, –1) (4) (3, 1)

1.21 If $x + y + 1 = 0$ and $x + 2y + 1 = 0$ are angle bisector of lines L_1 and L_2 and point (0, 0) lies on L_1, then acute angle bisector of L_1 and L_2 is
(1) $x + y - 1 = 0$ (2) $x + y + 1 = 0$ (3) $x + 2y + 1 = 0$ (4) data insufficient

1.22 Bisector of angle between $x + 2y - 1 = 0$ and $2x + y + 1 = 0$ containing (2, 1) is
(1) $x + y = 0$ (2) $x - y + 1 = 0$ (3) $x + y + 2 = 0$ (4) $x - y + 2 = 0$

1.23 Point where all lines of family $x(a + 2b) + y(a + 3b) = a + b$ are concurrent is
(1) (2, 1) (2) (2, –1) (3) (1, 2) (4) (1, –2)

1.24 Equation of line passing through point of intersection of $x + y + 2 = 0$ and $x - y + 4 = 0$ and having x-intercept = 0
(1) $x + 3y = 2$ (2) $3x + y = 0$ (3) $2x + y + 5 = 0$ (4) $x + 3y = 0$

1.25 If the slope of one line is double the other in line pair $x^2 + 6xy + by^2 = 0$, then b = ?
(1) 2 (2) 4 (3) 6 (4) 8

1.26 Equation of line pair perpendicular to $ax^2 + by^2 + 2hxy = 0$
(1) $ax^2 + 2hxy - by^2 = 0$ (2) $ax^2 - 2hxy + by^2 = 0$
(3) $bx^2 - 2hxy + ay^2 = 0$ (4) $bx^2 + 2hxy + 2y^2 = 0$

1.27 The sum of square of distances of a point from axes is 4 then its locus is
(1) $\sqrt{x^2+y^2} = 14$ (2) $\sqrt{x^2+y^2} = 12$ (3) $x^2 + y^2 = 4$ (4) $x^2 + y^2 = 16$

Level : II (Tough)

1.28 The point A(2, 1) is translated parallel to the line $x - y = 3$ by a distance 4 units. If its new position A′ is in third quadrant then the co-ordinates of A′ are
(1) $(2-2\sqrt{2}, 1-2\sqrt{2})$ (2) $(2+2\sqrt{2}, 2-2\sqrt{2})$ (3) $(-2+2\sqrt{2}, 2\sqrt{2}+1)$ (4) $(2\sqrt{2}-1, 2\sqrt{2}+2)$

1.29 Find point P on x-axis such that (AP + PB) is minimum where A(1, 1) & B(3, 4)
(1) $\left(\frac{7}{5}, 0\right)$ (2) $\left(\frac{9}{5}, 0\right)$ (3) $\left(\frac{6}{5}, 0\right)$ (4) (2, 0)

1.30 The distance of (0, 0) from $y = 2x + 2$ measured along $y = x$
(1) 1 (2) $\sqrt{2}$ (3) 2 (4) $\frac{1}{2}$

1.31 Values of α if $(\alpha, 2\alpha)$ lies inside the ΔABC if A(0, 2), B(2, 0) and C(4, 4)
(1) $\alpha \in \left(\frac{1}{3}, \frac{2}{3}\right)$ (2) $\alpha \in \left(\frac{2}{3}, 1\right)$ (3) $\alpha \in \left(\frac{2}{3}, \frac{4}{3}\right)$ (4) $\alpha \in \left(\frac{1}{3}, 1\right)$

1.32 Distance between lines represented by $x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$
(1) 1 (2) 2 (3) 3 (4) 4

1.33 If line joining (0, 0) to point of intersection of $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined with axis then k = ?
(1) 0 (2) 1 (3) – 1 (4) – 2

1.34 A variable line passing through fixed point (a, b) intersect the coordinate axes at A and B if O is origin, then locus of centroid of the triangle OAB is
(1) $bx + ay - 3xy = 0$ (2) $bx + ay - 2xy = 0$ (3) $ax + by - 3xy = 0$ (4) $ax + by - 2xy = 0$

1.35 On the portion of the straight line x + 2y = 4 intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co-ordinates :

(1) (2, 3) (2) (3, 2) (3) (3, 3) (4) none

1.36 If $P\left(1+\frac{t}{\sqrt{2}}, 2+\frac{t}{\sqrt{2}}\right)$ be any point on a line, then the range of values of t for which the point P lies between the parallel lines x + 2y = 1 and 2x + 4y = 15 is

(1) $-\frac{4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6}$ (2) $0 < t < \frac{5\sqrt{2}}{6}$ (3) $-\frac{4\sqrt{2}}{5} < t < 0$ (4) none of these

1.37 If vertices of Δ are (8, – 2), (2, – 2) & (8, 6), then find its orthocenter

(1) (8, – 2) (2) (8, 6) (3) (2, 2) (4) (–2, 2)

SECTION - II : ASSERTION & REASONING TYPE

1.38 **Statement-1 :** There is only one circle passing through (–3, 4), (2, 1) & (7, –2)

Statement-2 : Equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a circle if $\Delta \neq 0$, $a = b \neq 0$ and $h = 0$.

(1) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true ; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

1.39 **Statement-1 :** Area of ΔABC where A(20, 22), B(21, 24), C(22, 23) and area of ΔPQR where P(0, 0), Q(1, 2), R(2, 1) is equal

Statement-2 : The area of Δ be constant with respect to parallel transformation of co-ordinate axes.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
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(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

1.40 **Statement-1 :** If the middle point of the sides of a ΔABC are (0, 0), (1, 2), (– 3, 4) then centroid of ΔABC is $\left(\frac{-2}{3}, 2\right)$

Statement-2 : Centroid of a ΔABC and centroid of the triangle formed by joining the mid points of sides of ΔABC be always same

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1.41 **Statement-1 :** Two of the straight lines represented by the equation $ax^3 + bx^2y + cxy^2 + dy^3 = 0$ will be at right angle if $a^2 + ac + bd + d^2 = 0$

Statement-2 : If roots of equation $px^3 + qx^2 + rx + s = 0$ are α, β and γ, then $\alpha\beta\gamma = -s/p$.

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1.42 **Statement-1 :** The diagonals of the quadrilateral whose sides are $3x + 2y + 1 = 0$, $3x + 2y + 2 = 0$, $2x + 3y + 1 = 0$ and $2x + 3y + 2 = 0$ include an angle $\pi/2$

Statement-2 : Diagonals of a parallelogram bisect each other.

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1.43 **Statement-1 :** Each point on the line $y - x + 12 = 0$ is at same distance from the lines $3x + 4y - 12 = 0$ and $4x + 3y - 12 = 0$.

Statement-2 : locus of point which is at equal distance from the two given intersecting lines is the angle bisectors of the two lines.

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1.44 **Statement-1** Area of triangle formed by the line which is passing through the point (5, 6) such that segment of the line between axes is bisected at the point, with coordinate axes is 60 sq. units

Statement-2 : Area of triangle formed by line passing through point (α, β), with axes is maximum when point (α, β) is mid point of segment of line between axes.

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Statement-2 : locus of point which is at equal distance from the two given intersecting lines is the angle bisectors of the two lines.

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TOPIC 2 CIRCLE

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

2.1 Equation of chord of minimum length passing through (1, 2) of circle $x^2 + y^2 - 4x - 2y - 4 = 0$ is $ax + by + c = 0$ then $a + b + c =$

(1) 8 (2) – 8 (3) ±8 (4) None of these

2.2 Two circle of radii 8 & 6 intersect at right angle, then the length of common chord is :-

(1) $\frac{48}{\sqrt{5}}$ (2) $\frac{24}{\sqrt{5}}$ (3) $\frac{24}{5}$ (4) $\frac{48}{5}$

2.3 Consider the circles $x^2 + y^2 = 9$ and $x^2 + y^2 - 10x + 9 = 0$, then find the length of the common chord of circle is

(1) $\frac{6}{5}$ (2) $\frac{12}{5}$ (3) $\frac{24}{5}$ (4) 6

2.4 If length of the common chord of the circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ is λ, then the value of $[\lambda]$. (where [.] denotes greatest integer function)

(1) 1 (2) 2 (3) 3 (4) 4

2.5 The equation of circle which touches the line $x + 3y - 2 = 0$ and $3x + 9y - 2 = 0$, also the point of contact on first line is (– 1, 1), is

(1) $x^2 + y^2 - \frac{32x}{15} + \frac{8y}{5} + \frac{26}{15} = 0$ (2) $x^2 + y^2 + \frac{32x}{15} + \frac{8y}{5} + \frac{26}{15} = 0$

(3) $x^2 + y^2 + \frac{32x}{15} - \frac{8y}{5} + \frac{26}{15} = 0$ (4) None of these

2.6 P is a variable point on a circle with centre at C. CA and CB are perpendiculars from C to x and y-axis respectively. If the locus of the centroid of ΔPAB is a circle with centre (3, 6) and radius equal to 1, then the centre and radius of circle, whose centre is C, is.

(1) $\left(\frac{9}{2}, 9\right)$ & 3 (2) $\left(9, \frac{9}{2}\right)$ & 2 (3) $\left(\frac{9}{2}, \frac{9}{2}\right)$ & 3 (4) (9, 9), 3

2.7 If $A(2 + 3\cos\alpha, -3 + 3\sin\alpha)$, $B\left(2 + 3\cos\left(\alpha + \frac{2\pi}{3}\right), -3 + 3\sin\left(\alpha + \frac{2\pi}{3}\right)\right)$ and $C\left(2 + 3\cos\left(\alpha + \frac{4\pi}{3}\right), -3 + 3\sin\left(\alpha + \frac{4\pi}{3}\right)\right)$ be the angular points of a ΔABC then incentre of that triangle is

(1) (3, 2) (2) (0, 0) (3) (2, – 3) (4) (– 2, – 3)

2.8 The set of values of a for which the point (a – 1, a + 1) lies outside the circle $x^2 + y^2 = 8$ and inside the circle $x^2 + y^2 - 12x + 12y - 62 = 0$ is

(1) $(\sqrt{3}, 3\sqrt{2}) \cup (-\infty, 0)$ (2) $(-3\sqrt{2}, -\sqrt{3}) \cup (\sqrt{3}, 3\sqrt{2})$

(3) $(-3\sqrt{2}, -\sqrt{3}) \cup (0, \infty)$ (4) None of these

2.9 The range of parameter 'a' for which the variable line $y = 2x + a$ lies between the circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 16x - 2y + 61 = 0$ without intersecting either circle is

(1) $(-\infty, -15 - 2\sqrt{5})$ (2) $(-15 + 2\sqrt{5}, -\sqrt{5} - 1)$ (3) $(-15 + 2\sqrt{5}, \infty)$ (4) $(-15, -1)$

2.10 The point on the circle $x^2 + y^2 = a^2$ in first quadrant, so that tangent drawn at this point make a triangle of area a^2 with the coordinate axes, is

(1) $\left(\frac{3a}{\sqrt{2}}, \frac{3a}{\sqrt{2}}\right)$ (2) $\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$ (3) $\left(\frac{a}{2}, \frac{a}{2}\right)$ (4) None of these

2.11 The locus of the point from where tangents are drawn to the circle $x^2 + y^2 = 16$ and the product of the slopes of these tangents is 2, is

(1) $x^2 - 2y^2 = 16$ (2) $2x^2 - y^2 = 16$ (3) $x^2 - y^2 = 16$ (4) $2x^2 + y^2 = 16$

2.12 From a point on the line $4x - 3y = 6$ tangents are drawn to the circle $x^2 + y^2 - 6x - 4y + 4 = 0$ which make an angle of $\tan^{-1}\frac{24}{7}$ between them, then the coordinates of all such points are

(1) (– 2, 0), (6, – 6) (2) (2, 0), (6, 6) (3) (0,– 2) and (6, 6) (4) None of these

2.13 If the line $4x - 3y = -12$ is tangent at point (– 3, 0) and the line $3x + 4y = 16$ is tangent at the point (4, 1) to a circle, then equation of the circle is

(1) $(x - 1)^2 + (y - 3)^2 = 25$ (2) $(x - 1)^2 + (y + 3)^2 = 25$
(3) $(x + 1)^2 + (y - 3)^2 = 25$ (4) $(x - 1)^2 + (y - 2)^2 = 25$

2.14 If the chord of contact of tangents drawn from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$, then

(1) $2b = a + c$ (2) $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ (3) $b^2 = ac$ (4) None of these

2.15 If the two circles of radii 12 and 9 intersect each other at two distinct point orthogonally, then the distance between their centres is

(1) 15 (2) 16 (3) 18 (4) 13

2.16 If the polar of a point (α, β) with respect to any one of the circle $x^2 + y^2 - 2kx + 3 = 0$, where k is a variable, always passes through a fixed point, whatever be the value of k, then the fixed point is

(1) $\left(-\alpha, \frac{1}{\beta}(\alpha^2 - 3)\right)$ (2) $\left(\alpha - 3, \frac{\beta - 3}{\alpha - 3}\right)$ (3) $\left(-\alpha, \frac{1}{\beta}\right)$ (4) $(\alpha^2 + 3, \beta^2 - 3)$

2.17 The radius of inscribed circle in the quadrilateral formed by tangents drawn from (3, – 1) to circle $x^2 + y^2 - 2x - 2y - 2 = 0$ and radius formed by point of contact of these tangents is

(1) 1 (2) 2 (3) 3 (4) 3/2

2.18 If the locus of the mid-points of the chords of the circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 = a(x + y)$ subtends a right angle at origin is $x^2 + y^2 = 2(x + y)$, then the value of 'a' is

(1) 5 (2) – 2 (3) 2 (4) 3

2.19 If circles $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$, touch each other then

(1) $\frac{1}{a^2} + \frac{1}{b^2} = c^2$ (2) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ (3) $a^2 + b^2 = c^2$ (4) None of these

2.20 If $C_1 : x^2 + y^2 = r_1^2$ and $C_2 : (x - \alpha)^2 + (y - \beta)^2 = r^2$ be two circles with C_2 lying inside C_1 and touches C_1. Circle C lying inside C_1 touches C_1 internally and C_2 externally, then the locus of centre C is.

(1) $\sqrt{x^2 + y^2} = \sqrt{(x - \alpha)^2 + (y - \beta)^2}$ (2) $(x - \alpha)^2 + (y - \beta)^2 = x^2 + y^2$

(3) $\sqrt{x^2 + y^2} + \sqrt{(x - \alpha)^2 + (y - \beta)^2} = r_1 + r_2$ (4) None of these

2.21 If two circles pass through two points (0, a) and (0, – a) and touch the straight line y = mx + c will cut orthogonally, then
(1) $c^2 = a^2(2 + m^2)$ (2) $c^2 = -a^2(1 + m^2)$ (3) $c^2 = a^2(1 + m^2)$ (4) $c^2 = a^2(3 + 2m^2)$

2.22 The locus of the centre of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 4x + 6y + 4 = 0$ orthogonally is
(1) $8x + 12y - 5 = 0$ (2) $8x - 12y + 5 = 0$ (3) $8x + 12y + 61 = 0$ (4) $-8x - 12y + 13 = 0$

2.23 The locus of the point $(\sqrt{3h+2}, \sqrt{3k})$. If (h, k) lies on x + y = 1 is
(1) a pair of straight lines (2) a circle
(3) a parabola (4) an ellipse

2.24 The four points of intersection of the lines (2x – y + 1) (x – 2y + 3) = 0 with the axes lie on a circle whose centre is at the point
(1) $\left(-\frac{7}{4}, \frac{5}{4}\right)$ (2) $\left(\frac{3}{4}, \frac{5}{4}\right)$ (3) $\left(\frac{9}{4}, \frac{5}{4}\right)$ (4) $\left(0, \frac{5}{4}\right)$

2.25 α, β and γ are parametric angles of the points P, Q and R respectively, on the circle $x^2 + y^2 = 1$ and A is the point (– 1, 0). If the lengths of the chord AP, AQ and AR are in G.P. then $\cos\frac{\alpha}{2}$, $\cos\frac{\beta}{2}$ and $\cos\frac{\gamma}{2}$ are in
(1) A.P. (2) G.P. (3) H.P. (4) None of these

2.26 The radical centre of three circles, described on the three sides 3x – 2y + 10 = 0, x – y + 5 = 0 and 2x + 3y – 3 = 0 of a triangle as diameter, is
(1) $\left(\frac{24}{13}, \frac{29}{13}\right)$ (2) $\left(-\frac{24}{13}, \frac{29}{13}\right)$ (3) $\left(\frac{6}{13}, \frac{-5}{13}\right)$ (4) None of these

2.27 A circle of constant radius 4 passes through origin O, and cuts the axes at P and Q, then locus of the foot of the perpendicular from O to PQ is
(1) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{64}{(x^2+y^2)^2}$ (2) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{64}{x^2+y^2}$
(3) $x^2 + y^2 = \frac{16}{x^{-2}+y^{-2}}$ (4) None of these

2.28 From the point $P(2+3\sqrt{2}\cos\theta, 3+3\sqrt{2}\sin\theta)$, 0 < θ < 2π, tangents are drawn to the circle $x^2 + y^2 - 4x - 6y + 4 = 0$, then the angle between them is.
(1) $\frac{\pi}{5}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

2.29 A circle passes through the intersection points of $x^2 + y^2 - 8x = 0$ and $x^2 + y^2 = 9$ and the common chord of above circles is diameter of that circle then equation of circle is
(1) $x^2 + y^2 - 3x - 5 = 0$ (2) $16x^2 + 16y^2 - 36x - 207 = 0$
(3) $32x^2 + 32y^2 - 72x - 207 = 0$ (4) None of these

Level : II (Tough)

2.30 If $A\left(a, \frac{1}{a}\right), B\left(b, \frac{1}{b}\right), C\left(c, \frac{1}{c}\right)$ and $D\left(d, \frac{1}{d}\right)$ are 4 distinct points on a unit circle then abcd equals.
(1) 2 (2) 4 (3) 1 (4) 8

2.31 Let x, y be the real number satisfying the equation $x^2 + y^2 - 4x + 3 = 0$ Let maximum value of $x^2 + y^2$ be M and minimum value be m then M + m equals :
(1) 8 (2) 12 (3) 10 (4) 13

2.32 Two thin rods AB and CD of length 2a and 2b move along OX and OY respectively where O is the origin. The equation of locus of centre of circle passing through the extrimities of the two rods is :
(1) $x^2 - y^2 = a^2 - b^2$ (2) $x^2 + y^2 = a^2 - b^2$ (3) $x^2 + y^2 = a^2 + b^2$ (4) $x^2 - y^2 = a^2 + b^2$

2.33 The value of c for which the set $\{(x, y) \mid x^2 + y^2 + 2x - 1 \le 0\}$ and $\{(x, y) \mid x - y + c \ge 0\}$ contains only one point in common is :
(1) $(-\infty, -1) \cup [3, \infty]$ (2) $\{-1, 3\}$ (3) $\{-3\}$ (4) $\{-1\}$

2.34 If a circles $x^2 + y^2 + 2g_1x + 2f_1y = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each other, then
(1) $f_1g_1 = f_2g_2$ (2) $\frac{f_1}{g_1} = \frac{f_2}{g_2}$ (3) $f_1f_2 = g_1g_2$ (4) None of these

2.35 Two circle whose radii are equal to 4 and 8 intersect at right angles. Length of their common chord is :
(1) $\frac{16}{\sqrt{5}}$ (2) $\frac{8}{\sqrt{5}}$ (3) $4\sqrt{6}$ (4) 8

2.36 If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c \sin^2\alpha + (g^2 + f^2)\cos^2\alpha$ then the angle between the tangents is :
(1) α (2) 2α (3) $\frac{\alpha}{2}$ (4) $\frac{\pi}{2} - \alpha$

2.37 The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through some fixed point. One of them may be :
(1) $(1 + \sqrt{3}, 1)$ (2) $(1 + \sqrt{3}, 0)$ (3) $(1 - \sqrt{3}, -1)$ (4) $(1 + \sqrt{3}, 2)$

2.38 A circle whose centre lies in first quadrant passes through (3, 0) and cut off equal chords of length 4 units along the lines $x + y - 3 = 0$ and $x - y - 3 = 0$
(1) $x^2 + y^2 - 6y + 7 = 0$ (2) $x^2 + y^2 + 6y - 7 = 0$
(3) $x^2 + y^2 - 6x - 7 = 0$ (4) $x^2 + y^2 + 6x - 7 = 0$

2.39 Line $(x - 3)\cos\theta + (y - 3)\sin\theta = 1$ touches a circle $(x - 3)^2 + (y - 3)^2 = 1$, then find the number of values of θ.
(1) 1 (2) 2 (3) 3 (4) infinite

SECTION - II : ASSERTION & REASONING TYPE

2.40 **Statement-1 :** No tangent can be drawn from the point (1, 0) to the circle $x^2 + y^2 - 6x + 4y - 3 = 0$
Statement-2 : The power of the point of the circle $x^2 + y^2 + ax + by + c = 0$ with respect to point (a, b) is negative then point lies inside the circle
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

2.41 **Statement-1 :** Circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 7 = 0$ intersect each other at two distinct points
Statement-2 : Circles with centre C_1 and C_2 and radii r_1 and r_2 intersect at two distinct points, if $|C_1C_2| < r_1 + r_2$
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

2.42 **Statement-1 :** Number of circles through A(2, 4), B(5, 6), C(1, –2) is 1
Statement-2 : Through three non collinear points in a plane only one circle can be drawn.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

2.43 **Statement-1 :** The length of intercept made by the circle $x^2 + y^2 - 3x + 4y = 0$ on y-axis is 4.

Statement-2 : The length of y intercept of the circle.

$x^2 + y^2 + 2gx + 2fy + c = 0$ is $2\sqrt{g^2 - c}$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

2.44 **Statement-1 :** Three circles with non-collinear centres have exactly one circle cutting all the 3 circles orthogonally.

Statement-2 : Radical axis of intersecting circles is their common chord.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

2.45 **Statement-1 :** If L = 0 is tangent to circle S = 0, true willl be tangent to circle $S + \lambda L = 0$

Statement-2 : Perpendicular distance from centre of a circle to any its tangent is equal to radius of the circle.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

TOPIC 3 PARABOLA

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

3.1 A tangent to the parabola $y^2 + 4bx = 0$ meets the parabola $y^2 = 4ax$ at P and Q then the locus of middle point of PQ is :-
(1) $y^2(a + 2b) = 4a^2x$ (2) $y^2(2a + b) = 4b^2x$ (3) $y^2(2a + b) = 4a^2x$ (4) $y^2(a + 2b) = 4b^2x$

3.2 The normals at P, Q the ends of a focal chord of a parabola $y^2 = 4ax$ meets the parabola again in P' & Q' respectively then P'Q' =
(1) PQ (2) 2PQ (3) $\frac{1}{2}$PQ (4) 3PQ

3.3 If b and c are the lengths of the segments of any focal chord of a parabola $y^2 = 4ax$, then the length of the semi latus rectum is -
(1) $\frac{b+c}{2}$ (2) $\frac{bc}{b+c}$ (3) $\frac{2bc}{b+c}$ (4) $\sqrt{bc}$

3.4 If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ on the line $y - 2x = 1$, then a =
(1) 3 (2) –2 (3) –1 (4) 2

3.5 The values of a for which the point (–2a, a + 1) will be an interior point of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the parabola $y^2 = 4x$ is/are :-
(1) $[-1, -5 + 2\sqrt{6}]$ (2) $(-1, -5 + 2\sqrt{6})$ (3) (–1, 3/5) (4) $(-5 - 2\sqrt{6}, -5 + 2\sqrt{6})$

3.6 The equation of the parabola whose vertex is (–3, 0) and directrix is x + 5 = 0
(1) $y^2 = -8(x + 3)$ (2) $y^2 = 8(x + 3)$ (3) $x^2 = -8(y + 3)$ (4) $x^2 = 8(y + 3)$

3.7 Find length of focal chord of parabola $y^2 = 8x$ which is parendicular to line x + y = 1
(1) $4\sqrt{2}$ (2) $8\sqrt{2}$ (3) 4 (4) 8

3.8 Angle between the pair of tangents to $y^2 = 4a(x + a)$ drawn from (–2a, 2)
(1) 30° (2) 45° (3) 60° (4) 90°

3.9 If line y = 2x + c is a normal to the parabola $y^2 = 4x$ at the point $(t^2, 2t)$ then
(1) c = –12, t = –2 (2) c = 12, t = –2
(3) c = 12, t = 2 (4) c = –12, t = 2

3.10 A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 = 4(x - 1)$. After reflection the ray passes through
(1) (2, 0) (2) (1, 0) (3) (0, 0) (4) (–1, 0)

3.11 The double ordinate of parabola $y^2 = 8kx$ is of length 16k. The angle subtended by it at the vertex of parabola is -
(1) 45° (2) 60° (3) 90° (4) data insuffiicent

3.12 The equation of the parabola whose vertex & focus lie on the axis of x at distance d_1 & d_2 from the origin respectively
(1) $y^2 = 4(d_2 - d_1)x$ (2) $y^2 = 4(d_2 - d_1)(x - d_1)$
(3) $y^2 = 4(d_2 - d_1)(x - d_2)$ (4) $y^2 = 4(d_2 - d_1)(x + d_2)$

3.13 If end points t_1, t_2 of a chord satisfy the relation $t_1t_2 = 1$ then chord always passes through the point if parabola is $y^2 = 4x$
(1) (1, 0) (2) (–1, 0) (3) (2, 0) (4) (–2, 0)

3.14 Equation of common tangents of the parabola $y^2 = 4x$ and $x^2 = 4y$.
(1) $y = -x - 1$ (2) $y = -2x - 1$ (3) $y = -x - 2$ (4) $y = -x$

3.15 If Q_1 & Q_2 be the angle made by tangents to the axis of $y^2 = 4x$ from point P & if $Q_1 + Q_2 = 45°$ then locus of point P is
(1) $y = (1 - x)$ (2) $y = (2 - x)$ (3) $y = (x - 1)$ (4) $y = (x + 1)$

3.16 If $y = x + 1$ intersect the $x^2 = y$ at A & B then point of intersection of tangents at A & B is
(1) (–2, 4) (2) (–2, –4) (3) (2, 4) (4) $\left(\frac{1}{2}, 1\right)$

3.17 Locus of middle point of the chord of the parabola $y^2 = 4x$ which passes through a point (1, 2)
(1) $(y + 1)^2 = (x + 1)$ (2) $(y - 1)^2 = (2x - 1)$ (3) $(y + 1)^2 = (x - 1)$ (4) $(y - 1)^2 = (x + 1)$

3.18 Locus of mid point of chord of parabola $x^2 = 4y$ having slope 2
(1) $x = 2$ (2) $x = 4$ (3) $x = 6$ (4) $x = 0$

3.19 AB is a chord of the parabola $y^2 = 8x$ with vertex at A, BC is drawn perpendicular to AB meeting the axis at C then the projection of BC on the axis of parabola is
(1) 1 (2) 8 (3) 3 (4) 4

3.20 If M is the foot of the perpendicular from a point on a parabola $y^2 = 4x$ to its directrix & SPM is an equilateral triangle where S is the focus, then SP is equal to
(1) 1 (2) 2 (3) 3 (4) 4

3.21 Number of distinct normals that can be drawn from $\left(\frac{11}{4}, \frac{1}{4}\right)$ to the parabola $y^2 = 4x$ is
(1) 1 (2) 2 (3) 3 (4) 4

3.22 Line $\ell x + my + n = 0$ is tangent to parabola if
(1) $\ell n = am^2$ (2) $\ell m = an^2$ (3) $\ell n = am$ (4) $\ell m = an$

3.23 Length of focal chord of the parabola $y^2 = 4ax$ at a distance P from the vertex is
(1) $\frac{2a^2}{P}$ (2) $\frac{a^3}{P^2}$ (3) $\frac{4a^3}{P^2}$ (4) $\frac{P^2}{a}$

3.24 The extremities of a focal chord of a parabola $y^2 = 4ax$ are $P(t_1)$ and $Q(t_2)$ then which of the following is not correct about tangents at P and Q
(1) Are perpendicular (2) intersect on Directrix
(3) intersect at vertex (4) $t_1 t_2 = -1$

3.25 A circle with centre at the focus of the parabola $y^2 = 4ax$ touches the directrix then point of intersection of the circle and parabola is
(1) (–a, 2a) (2) (a, 3a) (3) (a, 2a) (4) (0, a)

3.26 If one end of focal chord of parabola $y^2 = 4x$ is (1, 2) then other end does not lie on
(1) $x^2y + 2 = 0$ (2) $xy = 2$ (3) $xy = -2$ (4) $x^2 + xy - y - 1 = 0$

3.27 $y^2 = 4ax$ be a parabola & $x^2 + y^2 + 2bx = 0$ be a circle, touch each other externally then :
(1) $a > 0, b > 0$ (2) $a > 0, b < 0$ (3) $a < 0, b > 0$ (4) $a < 0, b = 0$

3.28 If two normal drawn from any point to the parabola $y^2 = 4ax$ makes angle α & β with the axis such that $\tan \alpha . \tan \beta = 2$ then locus of this point is
(1) pair of straight line (2) circle (3) parabola (4) hyperbola

3.29 A tangent to the parabola $x^2 + 4ay = 0$ cuts the parabola $x^2 = 4by$ at A and B then locus of the mid point of AB is:
(1) $(a + 2b)\,x^2 = 4\,b^2y$ (2) $(b + 2a)\,x^2 = 4\,b^2y$ (3) $(a + 2b)\,y^2 = 4\,b^2x$ (4) $(b + 2x)\,x^2 = 4\,a^2y$

Level : II (Tough)

3.30 A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha, 0)$ $(\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is:

(1) $\sqrt{\frac{bc}{a}}$ (2) ac^2 (3) $\frac{b}{a}$ (4) $\sqrt{\frac{c}{a}}$

3.31 Number of normals drawn from the point $(-2, 2)$ to the parabola $y^2 - 2y - 2x - 1 = 0$ is

(1) one (2) two (3) three (4) zero

3.32 The directrix of the parabola $25[x^2 + y^2 - 2y + 1] = (3x + 4y + 1)^2$ is

(1) $3x + 4y + 1 = 0$ (2) $3x - 4y + 1 = 0$ (3) $4x + 3y + 1 = 0$ (4) $3x + 4y + 3 = 0$

3.33 If chord of parabola $y^2 = 4x$ subtend an angle of 90° at origin then it always passes through point

(1) (4, 0) (2) (0, 4) (3) (2, 0) (4) (1, 0)

3.34 Equation of common tangent to $x^2 + y^2 = 4$ and $y^2 = 4x$ having (–)ve slope

(1) $y = -\left(\sqrt{\frac{\sqrt{2}-1}{2}}\right)x + \sqrt{\frac{\sqrt{2}-1}{2}}$ (2) $y = -\left(\sqrt{\frac{\sqrt{2}-1}{2}}\right)x - \sqrt{\frac{2}{\sqrt{2}-1}}$

(3) $y = -(\sqrt{2} - 1)x + \left(\frac{1}{\sqrt{2}-1}\right)$ (4) $y = -(\sqrt{2} - 1)x - \left(\frac{1}{\sqrt{2}-1}\right)$

3.35 Locus of the point from where 3 normals are drawn to the parabola $y^2 = 4x$ such that two of them are perpendicular is

(1) $y^2 = (x - 1)$ (2) $y^2 = (x - 3)$ (3) $y^2 = (x + 3)$ (4) $y^2 = (x + 1)$

3.36 Two chords are drawn through a fixed point 't' on the parabola $y^2 = 4x$ at right angles. The chord joining there other extremities passes through a fixed point

(1) $[t^2 + 4, 2t]$ (2) $[t^2 + 4, -2t]$ (3) $[t^2 - 4, 2t]$ (4) $[t^2 - 4, -2t]$

3.37 If two normals to a parabola $y^2 = 4ax$ intersect at right angles then the chord joining their feet passes through a fixed point whose co-ordinates are:

(1) $(-2a, 0)$ (2) $(a, 0)$ (3) $(2a, 0)$ (4) none

3.38 The equation of the other normal to the parabola $y^2 = 4ax$ which passes through the point of intersection of normals at $(4a, -4a)$ & $(9a, -6a)$ is:

(1) $5x - y + 115a = 0$ (2) $5x + y - 135a = 0$

(3) $5x - y - 115a = 0$ (4) $5x + y + 115 = 0$

3.39 The co-ordinate of the vertex of parabola $y = x^2 + bx + c$ is (1, 2) then find value of b.

(1) –1 (2) –2 (3) 1 (4) 2

SECTION - II : ASSERTION & REASONING TYPE

3.40 **Statement-1 :** Curve $9y^2 - 6y = 2x + 1$ is symmetric about $y = \frac{1}{3}$

Statement-2 : A parabola is symmetric about it's axis.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

3.41 **Statement-1 :** Two perpendicular tangent on $y^2 = -4x$ is always meet on $x = 1$

Statement-2 : Two perpendicular tangents always meet on axis of parabola.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

3.42 **Statement-1 :** AB is a focal chord of a parabola then the tangent at A to the parabola is parallel to the normal at B.

Statement-2 : If $A(t_1)$ & $B(t_2)$ are the ends of a focal chord of the parabola $y^2 = 4ax$ then $t_1t_2 = -1$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

3.43 **Statement-1 :** The perpendicular bisector of the line segment joining the point $(-a, 2at)$ and $(a, 0)$ is tangent to the parabola $y^2 = 4ax$, where $t \in R$

Statement-2: Number of parabolas with a given point as vertex and length of latus rectum equal to 4, is 2.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

3.44 **Statement-1 :** Normal chord drawn at the point (8, 8) of the parabola $y^2 = 8x$ subtends a right angle at the vertex of the parabola.

Statement-2 : Every chord of the parabola $y^2 = 4ax$ passing through the point (4a, 0) subtends a right angle at the vertex of the parabola.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

3.45 Let $y^2 = 4ax$ be the parabola

Statement-1 : Circle circumscribing conormal points of a parabola passes through its vertex.

Statement-2 : If t_1, t_2, t_3 are feet of conormal points of the parabola, then $t_1 + t_2 + t_3 = 0$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 4

ELLIPSE

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

4.1 If the normal at the point P(θ) to the ellipse $5x^2 + 14y^2 = 70$ intersect it again at the point Q(3θ) and if $\cos 2\theta = \frac{a}{b}$ where a, b are coprime, then a – b =

(1) 5 (2) – 5 (3) 23 (4) –23

4.2 If latus rectum of the ellipse $x^2 \tan^2 \alpha + y^2 \sec^2 \alpha = 1$ is $\frac{1}{2}$ then $\alpha(0 < \alpha < \pi)$ is equal to :-

(1) $\frac{\pi}{12}$ (2) $\frac{\pi}{6}$ (3) $\frac{7\pi}{12}$ (4) $\frac{5\pi}{6}$

4.3 If θ is the angle between the diameter through any point on the ellipse $\frac{x^2}{64} + \frac{y^2}{49} = 1$ and the normal at that point then the greatest value of tan θ is :-

(1) $\frac{112}{15}$ (2) $\frac{15}{112}$ (3) $\frac{15}{56}$ (4) $\frac{56}{15}$

4.4 Find equation of ellipse having focus (–1, 1) & directrix y = 3 & eccentricity is $\frac{1}{2}$

(1) $4x^2 + 3y^2 + 8x - 2y + 1 = 0$ (2) $4x^2 + 3y^2 + 8x - 2y - 1 = 0$
(3) $3x^2 + 4y^2 + 8x - 2y + 1 = 0$ (4) $3x^2 + 4y^2 + 8x - 2y - 1 = 0$

4.5 Equation of ellipse having center at origin, axes are the axes of co-ordinate and passing through (2, 0) & $\left(1, \frac{\sqrt{3}}{2}\right)$

(1) $4x^2 + y^2 = 4$ (2) $x^2 - 4y^2 = 4$ (3) $x^2 + 4y^2 = 4$ (4) $4x^2 - y^2 = 4$

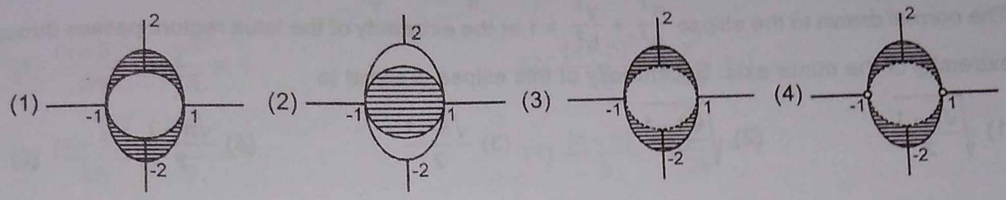
4.6 If minor-axis of ellipse subtend a right angle at its focus then eccentricity of ellipse is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\sqrt{2}$ (4) $\sqrt{3}$

4.7 Angle between diameter of ellipse having eccentric angle 30° & 120° if ellipse is $\frac{x^2}{9} + \frac{y^2}{16} = 1$

(1) $\tan^{-1}\left(\frac{48\sqrt{3}}{19}\right)$ (2) $\tan^{-1}\left(\frac{12\sqrt{3}}{19}\right)$ (3) $\tan^{-1}\left(\frac{24\sqrt{3}}{19}\right)$ (4) $\tan^{-1}\left(\frac{6\sqrt{3}}{19}\right)$

4.8 Draw the region bounded by $x^2 + y^2 > 1$ and $\frac{x^2}{1} + \frac{y^2}{4} \le 1$



4.9 The area of triangle formed by co-ordinate axis & tangent to ellipse $3x^2 + 4y^2 = 12$ which is parallel to $y = x + 1$ is
(1) 4 sq. unit (2) 6 sq. unit (3) 8 sq. unit (4) 10 sq. unit

4.10 Locus of foot of perpendicular drawn from centre to any tangent to the ellipse $x^2 + 2y^2 = 2$
(1) $(x^2 + y^2)^2 = (2x^2 - y^2)$ (2) $(x^2 + y^2)^2 = (2x^2 + y^2)$
(3) $(x^2 - y^2)^2 = (2x^2 - y^2)$ (4) $(x^2 - y^2) = (2x^2 + y^2)$

4.11 Shortest distance between the line $x + y = 10$ & ellipse $9x^2 + 16y^2 = 144$
(1) 5 (2) $5\sqrt{2}$ (3) $5/\sqrt{2}$ (4) 5/2

4.12 An ellipse slides between two perpendicular lines then locus of its center will be
(1) ellipse (2) parabola (3) circle (4) hyperbola

4.13 If tangents to $y^2 = 4x$ intersect the $\frac{x^2}{2} + y^2 = 1$ at A & B then find the locus of point of intersection of tangents at A & B.
(1) $2x^2 + y = 0$ (2) $2x^2 - y = 0$ (3) $2y^2 + x = 0$ (4) $2y^2 - x = 0$

4.14 Tangents are drawn from the points on the line $x = 5$ to $x^2 + 4y^2 = 4$ then all the chords of contact pass through a fixed point.
(1) $\left(\frac{4}{5}, 2\right)$ (2) $\left(\frac{4}{5}, 0\right)$ (3) $\left(\frac{-4}{5}, -2\right)$ (4) $\left(\frac{-4}{5}, 2\right)$

4.15 If $\sqrt{3}(bx) + ay = 2ab$ touched the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then eccentric angle is
(1) 30° (2) 45° (3) 60° (4) 90°

4.16 A line segment of length (a + b) moves in such a way that its ends are on two perpendicular straight lines. Then the locus of the point on this line which divides it into portions of lengths a & b is
(1) parabola (2) circle (3) ellipse (4) none

4.17 If chord of contact of the tangents drawn from the point (α, β) to $x^2 + 2y^2 = 2$, touches the circle $x^2 + y^2 = 1$ then locus of (α, β) is
(1) $4x^2 + y^2 = 4$ (2) $4x^2 - y^2 = 4$ (3) $x^2 - 4y^2 = 4$ (4) $x^2 + 4y^2 = 4$

4.18 If $\tan\theta_1 \tan\theta_2 = -2$ then the chord joining two points θ_1 & θ_2 on the ellipse $x^2 + 2y^2 = 2$ will subtend a right angle at
(1) focus (2) center (3) ends of major axis (4) ends of minor axis

4.19 If line $3x + 4y = -\sqrt{7}$ touches the ellipse $3x^2 + 4y^2 = 1$ then the point of contact is
(1) $\left(\frac{1}{\sqrt{7}}, \frac{1}{\sqrt{7}}\right)$ (2) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ (3) $\left(\frac{1}{\sqrt{7}}, \frac{-1}{\sqrt{7}}\right)$ (4) $\left(\frac{-1}{\sqrt{7}}, \frac{-1}{\sqrt{7}}\right)$

4.20 The equation of chord of ellipse $2x^2 + 5y^2 = 20$ which is bisected at the point (2, 1) is
(1) $4x + 5y + 13 = 0$ (2) $4x + 5y = 13$ (3) $5x + 4y = 13$ (4) $5x + 4y + 13 = 0$

4.21 The normal drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the extremity of the latus rectum passes through the extremity of the minor axis. Eccentricity of this ellipse is equal to
(1) $\sqrt{\frac{\sqrt{5}-1}{2}}$ (2) $\sqrt{\frac{\sqrt{3}-1}{2}}$ (3) $\frac{\sqrt{3}-1}{2}$ (4) $\frac{\sqrt{5}-1}{2}$

4.22 The equation $\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0$ represent the ellipse if
(1) $r > 2$ (2) $2 < r < 5$ (3) $r > 5$ (4) $r \in (2, 5)$

4.23 The curve $x = 3(\cos t + \sin t)$ $y = 4(\cos t - \sin t)$ represents
(1) ellipse (2) parabola (3) hyperbola (4) circle

4.24 Let P be the variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with focii at S & S′. Then maximum area of $\Delta PSS'$
(1) 24 (2) 12 (3) 36 (4) 48

4.25 If $y = x + 1$ is a polar to $x^2 + 2y^2 = 1$ then pole is
(1) $\left(-1, \frac{1}{2}\right)$ (2) $(-1, 1)$ (3) $\left(-\frac{1}{2}, 1\right)$ (4) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

4.26 The equation $\sqrt{(x-2)^2 + y^2} + \sqrt{(x+2)^2 + y^2} = 4$ represents
(1) line segment (2) parabola (3) ellipse (4) hyperbola

Level : II (Tough)

4.27 The locus of the point of intersection of tangents to an ellipse at two points whose eccentric angle differ by a constant α is $\frac{x^2}{\lambda \sec^2\left(\frac{\alpha}{2}\right)} + \frac{y^2}{\mu \sec^2\left(\frac{\alpha}{2}\right)} = 1$ then $\lambda + \mu$
(1) $a^2 + b^2$ (2) $a + b$ (3) $a^2 - b^2$ (4) $a \times b$

4.28 The locus of foot of perpendicular drawn from centre to any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
(1) $(x^2 + y^2)^2 = a^2x^2 - b^2y^2$ (2) $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$
(3) $(x^2 + y^2) = a^2x^2 + b^2y^2$ (4) $(x^2 - y^2)^2 = a^2x^2 + b^2y^2$

4.29 The condition on 'a' and 'b' for which two distinct chords of the ellipse $\frac{x^2}{2a^2} + \frac{y^2}{2b^2} = 1$ passing through (a, – b) are bisected by the line x + y = b is
(1) $a^2 < 7b^2 - 6ab$ (2) $a^2 > 7b^2 + 6ab$ (3) $a^2 > 7b^2 - 6ab$ (4) $a^2 < 7b^2 + 6ab$

4.30 Coordinates of the vertices B and C of a triangle ABC are (2, 0) and (8, 0) respectively. The vertex A is varying in such a way that $4\tan\frac{B}{2} \cdot \tan\frac{C}{2} = 1$. Then locus of A is

(1) $\frac{(x-5)^2}{25} + \frac{y^2}{16} = 1$ (2) $\frac{(x-5)^2}{16} + \frac{y^2}{25} = 1$

(3) $\frac{(x-5)^2}{25} + \frac{y^2}{9} = 1$ (4) $\frac{(x-5)^2}{9} + \frac{y^2}{25} = 1$

4.31 A circle has the same centre as an ellipse & passes through the focii F_1 & F_2 of the ellipse, such that the two curves intersect at 4 points. Let 'P' be any one of their points of intersection. If the major axis of the ellipse is 17 & the area of the triangle PF_1F_2 is 30, then the distance between the focii is :
(1) 11 (2) 12 (3) 13 (4) 15

4.32 Point 'O' is the centre of the ellipse with major axis AB & minor axis CD. Point F is one focus of the ellipse. If OF = 6 & the diameter of the inscribed circle of triangle OCF is 2, then the product (AB) (CD) is
(1) 64 (2) 12 (3) 65 (4) 3

4.33 If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with vertices having eccentric angles α, β, γ respectively is (x_1, y_1), then $\Sigma \cos\alpha \cos\beta + \Sigma \sin\alpha \sin\beta =$

(1) $\frac{9x_1^2}{a^2} + \frac{9y_1^2}{b^2} + \frac{3}{2}$ (2) $9x_1^2 - 9y_1^2 + a^2b^2$ (3) $\frac{9x_1^2}{a} + \frac{9y_1^2}{b} + 3$ (4) $\frac{9x_1^2}{2a^2} + \frac{9y_1^2}{2b^2} - \frac{3}{2}$

4.34 The locus of extremities of latus rectum of the family of ellipse $b^2x^2 + y^2 = a^2b^2$ where b is a parameter $(b^2 < 1)$, is-
(1) $x^2 \pm a^2y^2 = a^2$ (2) $x^2 \pm ay = a^2$ (3) $x \pm ay^2 = a^2$ (4) none of these

4.35 $\frac{(3x-4y+10)^2}{2} + \frac{(4x+3y-15)^2}{3} = 1$ is an ellipse, then find major & minor axes

(1) 6 and 4 (2) 150 and 100 (3) $10\sqrt{3}$ and $10\sqrt{2}$ (4) $\frac{2\sqrt{3}}{5}$ and $\frac{2\sqrt{2}}{5}$

SECTION - II : ASSERTION & REASONING TYPE

4.36 **Statement-1 :** : The minimum length of a tangent to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intercept between the coordinate axes is a + b

Statement-2 : : For each pair of two negative real numbers a and b, inequality $\frac{a+b}{2} \geq -\sqrt{ab}$ holds.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

4.37 **Statement-1 :** If F_1 and F_2 are the feet of the perpendiculars from foci S_1 & S_2 of an ellipse $\frac{x^2}{5} + \frac{y^2}{3} = 1$ on the tangent at any point P on the ellipse then $(S_1F_1).(S_2F_2) = 3$

Statement-2 : For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ the product of length of feet of perpendicular from focii to any tangent is equal to b^2

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 is **NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

4.38 **Statement-1 :** the portion of the tangent to $\frac{x^2}{9} + \frac{y^2}{4} = 1$ between the point of contact & the directrix subtends an angle of 90° at (0, 0)

Statement-2 : The portion of the tangent to an ellipse between the point of contact & the directrix subtends a right angle at the corresponding focus.

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 is **NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

4.39 **Statement -1 :** The circle on any focal distance as diameter touches the director circle.

Statement - 2 : If P be any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a < b)$ with S & S′ as its focii, then $\ell(SP) + \ell(S'P) = 2b$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

4.40 **Statement -1 :** An ellipse with major axis 4 and minor axis 2 touches both the coordinate axes. Locus of its focus is $(x^2 + y^2)(1 + x^2y^2) = 16x^2y^2$.

Statement - 2 : Locus of point of intersection of tangents at extremeties of a focal chord is the corresponding directrix.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

4.41 **Statement -1 :** An ellipse with major axis 4 and minor axis 2 touches both the coordinate axis. Locus of its centre is the circle $x^2 + y^2 = 5$

Statement-2 : Director circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 + b^2$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 5 HYPERBOLA

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

5.1 The locus of the middle points of the chords of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, which pass through a fixed point (α, β) is a hyperbola whose centre is :-

(1) $(2\alpha, 2\beta)$ (2) (α, β) (3) $(3\alpha, 3\beta)$ (4) $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

5.2 The locus of the points whose chord of contact with respect to the parabola $y^2 = 4ax$ touches the hyperbola $x^2 - y^2 = a^2$ is :-

(1) Pair of straight lines (2) Parabola (3) Circle (4) Ellipse

5.3 If a line $ax + by + c = 0$ is a normal to the hyperbola $xy = 1$ then

(1) $a > 0, b > 0$ (2) $a > 0, b < 0$ (3) $a < 0, b < 0$ (4) $a + b = 0$

5.4 A variable line $x \cos\alpha + y \sin \alpha = p$ which is a chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $(b > a)$ subtends a right angle at the centre of the hyperbola, then it always touches a fixed circle whose radius is :-

(1) $\frac{ab}{\sqrt{a^2+b^2}}$ (2) $\frac{ab}{\sqrt{b^2-a^2}}$ (3) $\frac{ab}{\sqrt{a^2-b^2}}$ (4) $\frac{ab}{2\sqrt{b^2-a^2}}$

5.5 The eccentricity of a hyperbola, the angle between whose asymptotes is 30°, is

(1) $\sqrt{6}$ (2) $\sqrt{2}$ (3) $\sqrt{6}-\sqrt{2}$ (4) $\sqrt{3}$

5.6 Eccentricity of standered hyperbola whose latus ractum is half of its transverse axis

(1) $e = \sqrt{3}$ (2) $e = \sqrt{3}/2$ (3) $e = \sqrt{2}$ (4) $e = \sqrt{\frac{3}{2}}$

5.7 Equation of hyperbola whose foci are (6, 0) & (–6, 0) and eccentricity = 2

(1) $3y^2 - x^2 = 27$ (2) $3x^2 - y^2 = 27$ (3) $x^2 - 3y^2 = 27$ (4) $x^2 - 3y^2 = 9$

5.8 Equation of hyperbola if its foci coicide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ & $e = 2$

(1) $x^2 - 3y^2 = 12$ (2) $2x^2 - 3y^2 = 12$ (3) $3x^2 - y^2 = 12$ (4) $3y^2 - x^2 = 12$

5.9 Which of the following pair may represent the eccentricities of two conjugate hyperbola for all $\alpha \in (0°, 90°)$

(1) $\sin\alpha, \cos\alpha$ (2) $\tan\alpha, \cot\alpha$ (3) $\sec\alpha, \operatorname{cosec}\alpha$ (4) $1 + \sin\alpha, 1 + \cos(\alpha)$

5.10 If P[$\sqrt{2}\tan\theta, \sqrt{2}\sec\theta$] is any point in Ist quadrent at distance $\sqrt{6}$ from origin then θ =

(1) 30° (2) 45° (3) 60° (4) 90°

5.11 The locus of the middle points of chord of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is

(1) $3x - 4y = 4$ (2) $3y - 4x + 4 = 0$ (3) $4x - 3y = 3$ (4) $3x - 4y = 2$

5.12 The tangents from $(1, 2\sqrt{2})$ to the hyperbola $16x^2 - 25y^2 = 400$ include an angle between them is equal to
(1) 30° (2) 45° (3) 60° (4) 90°

5.13 The co-ordinate of focus of $9x^2 - 16y^2 + 18x + 32y - 151 = 0$
(1) (–6, 1) (2) (6, 1) (3) (–4, 1) (4) (4, –1)

5.14 If the chord of contact of tangents from two points (x_1, y_1) & (x_2, y_2) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are at right angles then $\frac{x_1x_2}{y_1y_2} = ?$
(1) $-a^2/b^2$ (2) $-b^2/a^2$ (3) $-b^4/a^4$ (4) $-a^4/b^4$

5.15 If e & e′ are the eccentricities of the ellipse $5x^2 + 9y^2 = 45$ & hyperbola $5x^2 - 4y^2 = 45$ then ee′ = ?
(1) –1 (2) 1 (3) –4 (4) 9

5.16 If the length of the transverse & conjugate axes of a hyperbola be 8 & 6 respectively then the difference of focal distances of any point of the hyperbola will be
(1) 8 (2) 6 (3) 14 (4) 2

5.17 If m is a variable, then locus of the point of intersection of the lines $\frac{x}{3} - \frac{y}{2} = m$ & $\frac{x}{3} + \frac{y}{2} = \frac{1}{m}$ is
(1) parabola (2) ellipse (3) hyperbola (4) none

5.18 The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a rectangular hyperbola if
(1) $\Delta \neq 0, h^2 > ab, a + b = 0$ (2) $\Delta \neq 0, h^2 < ab, a + b = 0$
(3) $\Delta \neq 0, h^2 = ab, a + b = 0$ (4) none

5.19 If e & e′ be the eccentricity of two conics S & S′ such that $e^2 + (e')^2 = 3$ then both S & S′ are
(1) ellipse (2) parabola (3) hyperbola (4) none

5.20 The equation of normal at $(a \sec\theta_1, b \tan\theta)$ of the curve $b^2x^2 - a^2y^2 = a^2b^2$ is
(1) $ax \sec\theta + by \operatorname{cosec}\theta = a^2 + b^2$ (2) $ax \cot\theta + by \cos\theta = a^2 + b^2$
(3) $ax \cos\theta + by \cot\theta = a^2 + b^2$ (4) $ax \cos\theta + by \cot\theta = a^2 - b^2$

5.21 If PQ & PR are tangents drawn from a point P to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ if equation of QR is $4x - 3y - 6 = 0$ then P is
(1) (2, 6) (2) (3, 4) (3) (6, 2) (4) none

5.22 A tangent to a $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ intercepts a length of unity from each of the co-ordinate axes then the point (a, b) lies on
(1) $x^2 - y^2 = 2$ (2) $x^2 - y^2 = 1$ (3) $x^2 - y^2 = -1$ (4) none

5.23 The area of quadrilateral formed by focii of hyperbola $\frac{x^2}{4} - \frac{y^2}{3} = 1$ & its conjugate hyperbola is
(1) 14 (2) 24 (3) 12 (4) 10

5.24 If the focii of the ellipse $\frac{x^2}{k^2a^2} + \frac{y^2}{a^2} = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$ coincides then k = ?
(1) $\pm\sqrt{3}$ (2) $\pm\sqrt{2}$ (3) $\sqrt{3}$ (4) $\sqrt{2}$

5.25 If LL′ be the latus rectum through the focus S of a hyperbola and A be the farther vertex of the conic. If $\Delta ALL'$ is equilateral then its eccentricity is

(1) $\sqrt{3}$ (2) $\sqrt{3}+1$ (3) $(\sqrt{3}+1)/\sqrt{2}$ (4) $(\sqrt{3}+1)/\sqrt{3}$

5.26 The tangent at point $(2\sec\theta, 3\tan\theta)$ of the $\frac{x^2}{4}-\frac{y^2}{9}=1$ is parallel to $3x-y+4=0$ then the value of θ is

(1) 30° (2) 45° (3) 60° (4) 90°

Level : II (Tough)

5.27 The points of intersection of the curves whose parametric equations are $x=t^2+1$, $y=2t$ and $x=2s$, $y=2/s$ is given by :

(1) (4, 1) (2) (2, 2) (3) (–2, 2) (4) (1, 0)

5.28 From any point on the hyperbola $H_1 : \frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ tangents are drawn to the hyperbola $H_2 : \frac{x^2}{a^2}-\frac{y^2}{b^2}=2$. The area cut-off by the chord of contact on the asymptotes of H_2 is equal to:

(1) ab (2) 2ab (3) 4ab (4) 8ab

5.29 The equations of the transverse and conjugate axes of a hyperbola are respectively $x+2y-3=0$, $2x-y+4=0$, and their respective lengths are $\sqrt{2}$ and $2/\sqrt{3}$. The equation of the hyperbola is

(1) $\frac{2}{5}(x+2y-3)^2-\frac{3}{5}(2x-y+4)^2=1$ (2) $\frac{2}{5}(2x-y+4)^2-\frac{3}{5}(x+2y-3)^2=1$

(3) $2(2x-y+4)^2-3(x+2y-3)^2=1$ (4) $2(x+2y-3)^2-3(2x-y+4)^2=1$

5.30 The chord PQ of the rectangular hyperbola $xy=a^2$ meets the x-axis at A; C is the mid point of PQ & 'O' is the origin. Then the Δ ACO is :

(1) equilateral (2) isosceles (3) right angled (4) right isosceles.

5.31 If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB (O is the origin) is an equilateral triangle, then the eccentricity 'e' of the hyperbola satisfies

(1) $e>\sqrt{3}$ (2) $1<e<\frac{2}{\sqrt{3}}$ (3) $e=\frac{2}{\sqrt{3}}$ (4) $e>\frac{2}{\sqrt{3}}$

5.32 If $x\cos\alpha+y\sin\alpha=p$, a variable chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=1$ subtends a right angle at the centre of the hyperbola, then the chords touch a fixed circle whose radius is equal to

(1) $\sqrt{2}\,a$ (2) $\sqrt{3}\,a$ (3) 2 a (4) $\sqrt{5}\,a$

5.33 Consider the point P(4, 3) and $\frac{x^2}{16}-\frac{y^2}{9}=1$. Which of the following is not true ?

(1) Two tangents can be drawn from P (2) One tangent can be drawn from P
(3) P lies on out side the hyperbola (4) P lines on Asymptotes on Hyperbola

5.34 The hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and point P(4, 3), then find number of tangents from P.

(1) 0 (2) 1 (3) 2 (4) 3

SECTION - II : ASSERTION & REASONING TYPE

5.35 **Statement-1 :** The conic $16x^2 - 3y^2 - 32x + 12y - 4y = 0$ represent a hyperbola

Statement-2 : If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represent a hyperbola then $\Delta \neq 0$ and $h^2 - ab > 0$

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1

(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1

(3) Statement -1 is True, Statement -2 is False

(4) Statement -1 is False, Statement -2 is True

5.36 **Statement-1 :** The latus rectum of the hyperbola $x^2 - y^2 = a^2$ is equal to the length of its major axis

Statement-2 : The semi latus rectum of ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is equal to $\frac{b^2}{a}$

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1

(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1

(3) Statement -1 is True, Statement -2 is False

(4) Statement -1 is False, Statement -2 is True

5.37 Normal is drawn to the hyperbola $xy = c^2$ at point P (t_1) meets the hyperbola again at Q (t_2).

Statement-1 : Square of distance between P and Q is $c^2 (t_1 - t_2)^2 \left(1 + \frac{1}{t_1^2 t_2^2}\right)$.

Statement-2 : $t_1^3 t_2 = -1$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

TOPIC 6 SET & RELATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

6.1 Consider the following relations :
(1) $A - B = A - (A \cap B)$
(2) $A = (A \cap B) \cup (A - B)$
(3) $A - (A \cup C) = (A - B) \cup (A - C)$, where A,B,C are sets.
Which of these is correct ?
(1) 1 and 3 (2) 2 only (3) 2 and 3 (4) 1 and 2

6.2 Let U be the universal set and $A \cup B \cup C = U$. Then $\{(A - B) \cup (B - C) \cup (C - A)\}'$ is equal to
(1) $A \cup B \cup C$ (2) $A \cup (B \cap C)$ (3) $A \cap B \cap C$ (4) $A \cap (B \cup C)$

6.3 Which of the following is a function from A to B (where A = {1, 2, 5} and B = {a, b, c, d})
(1) {(1, a), (2, c), (1, d), (5, b)} (2) {(1, a), (2, c)}
(3) {(1, d), (2, b), (5, c)} (4) none of these

6.4 Let a relation R on the set N of natural number be defined as $(x, y) \in R$ if and only if $x^2 - 4xy + 3y^2 = 0$ for all $x, y \in N$. The relation R is -
(1) reflexive (2) symmetric (3) transitive (4) an equivalance relation

6.5 R is relation defined on R × R by (a, b) R (c, d) if a – c is an integer and b = d. The relation R is -
(1) an identity relation (2) an universal relation
(3) an equivalence relation (4) None of these

6.6 Let R be a relation from N to N defined by
$R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$ then the statement which is not true is
(1) $(a, b) \in R \ \forall \ a \in N$ (2) R is symmetric (3) R is transitive (4) all of these

6.7 The set {0, 2, 6, 12, 20} in the set-builder form is
(1) $\{x : x = n^2 - 3n + 2$, where n is a natural number & $1 \le n \le 5\}$
(2) $\{x : x = n^2 - 3n + 2$, where 'n' is a natural number & $1 \le n \le 6\}$
(3) $\{x : x = n^2 - 3n + 4$, where 'n' is a natural number & $1 \le n \le 5\}$
(4) $\{x : x = n^2 + 5n - 6$, where n is a natural number & $1 \le n \le 5\}$

6.8 For the set A = {a, b, c, d, e} the correct statement is
(1) $\{a, b\} \in A$ (2) $\{a\} \in A$ (3) $a \in A$ (4) $a \notin A$

6.9 If $A = \{x : x$ is an integer, $x^2 \le 1\}$, then the elements of 'A' are
(1) {0, 1} (2) {– 1, 0, 1} (3) {– 1, 0} (4) none of these

6.10 The solution set of the equation $x^3 - 3x + 2 = 0$ in roster form is
(1) {1, – 2} (2) {1, 2} (3) {1, 2, 3} (4) {– 1, 2}

6.11 The set equivalent to the set {1, 2, 3, 4, 5, 6} is
(1) {1, 2, 4, 5, 6}
(2) {3, 4, 1, 5, 2, 6}
(3) $\{x : x = n$, where n is natural number and $n \le 7\}$
(4) {6, 5, 4, 3, 2}

6.12 The number of proper subsets of the set {1, 2, 3} is
(1) 8 (2) 7 (3) 6 (4) 5

6.13 If A and B are two given sets, then $A \cup (A \cap B)$ is equal to
(1) A (2) B (3) A^c (4) B^c

6.14 If A and B are two sets then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to
(1) $A \cup B$ (2) $A \cap B$ (3) A (4) B'

6.15 If A, B and C are any three sets, then $A - (B \cap C)$ is equal to
(1) $(A - B) \cap (A - C)$ (2) $(A - B) \cup (A - C)$ (3) $(A - B) \cup C$ (4) $(A - B) \cap C$

6.16 If A = {x : x is a multiple of 3} and B = {x : x is a multiple of 5}, then A – B is ($\bar{A}$ means complement of A)
(1) $\bar{A} \cap B$ (2) $A \cap \bar{B}$ (3) $\bar{A} \cap \bar{B}$ (4) $\overline{A \cap B}$

6.17 Let A = {1, 2, 3, 4, 5} ; B = {2, 3, 6, 7}. Then the number of elements in $(A \times B) \cap (B \times A)$ is
(1) 18 (2) 6 (3) 4 (4) 0

6.18 Let A = {1, 2, 3}. The total number of distinct relations that can be defined over A is
(1) 2^9 (2) 6 (3) 8 (4) None of these

6.19 Let A = {1, 2, 3}, B = {1, 3, 5}. A relation $R : A \to B$ is defined by R = {(1, 3), (1, 5), (2, 1)}. Then R^{-1} is defined by
(1) {(1, 2), (3, 1), (1, 3), (1, 5)} (2) {(1, 2), (3, 1), (2, 1)}
(3) {(1, 2), (5, 1), (3, 1)} (4) None of these

6.20 Let R be a relation on the set N be defined by {(x, y) |x, y ∈ N, 2x + y = 41}. Then R is
(1) Reflexive (2) Symmetric (3) Transitive (4) None of these

6.21 Let R be the relation on the set R of all real numbers defined by aRb iff $|a - b| \le 1$. Then R is
(1) Reflexive and Symmetric (2) Symmetric only
(3) Transitive only (4) Anti-symmetric only

6.22 In a class of 30 pupils, 12 take needle work, 16 take physics and 18 take history. If all the 30 students take at least one subject and no one takes all three then the number of pupils taking 2 subjects is
(1) 16 (2) 6 (3) 8 (4) 20

6.23 The number of elements in the power set of {a, b, c}
(1) 3 (2) 7 (3) 8 (4) 6

6.24 The empty set of the following is /are
(1) {x : x is a real number and $x^2 - 1 = 0$} (2) {x : x is a real number and $x^2 + 1 = 0$}
(3) {x : x is a natural number and $2 \le x \le 3$} (4) {x : x is a real number and $x^2 = x + 2$}

6.25 The set A = {x : x ∈ R, $x^2 = 16$ and 2x = 6} equals
(1) {– 4, 3, 4} (2) ϕ (3) {–4, 4} (4) {3}

6.26 Let R = {(a, a)} be a relation on a set A. Then R is
(1) Symmetric (2) Antisymmetric
(3) Transitive (4) Neither symmetric nor anti-symmetric

6.27 If R is an equivalence relation on a set A, then R^{-1} is NOT
(1) Reflexive (2) Symmetric (3) Transitive (4) None of these

Level : II (Tough)

6.28 In a town of 10,000 families it was found that 40% family buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy A only is
(1) 3100 (2) 3300 (3) 2900 (4) 1400

6.29 The number of elements in the set {(a, b) : $2a^2 + 3b^2 = 35$, a, b ∈ Z), where Z is the set of all integers, is
(1) 2 (2) 4 (3) 8 (4) 12

6.30 Let a relation R be defined by R = {(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)}, then R^{-1} oR is

(1) Reflexive only (2) Symmetric only (3) Transitive only (4) Equivalence

6.31 Let R be a relation over the set N × N and it is defined by (a, b)R(c, d) ⇒ a + d = b + c. Then R is NOT

(1) Reflexive (2) Symmetric (3) Transitive (4) Anti Symmetric

6.32 Let R_1 be a relation defined by $R_1 = \{(a, b) \mid a \geq b, a, b \in R\}$. Then R_1 is

(1) Only reflexive (2) Both Reflexive and transitive

(3) Symmetric, transitive but not reflexive (4) Neither transitive nor reflexive but symmetric

6.33 Let A = {1, 2, 3, 4} and R be a relation in A given by R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)}. Then R is

(1) Reflexive (2) Anti symmetric (3) Transitive (4) An equivalence relation

SECTION - II : ASSERTION & REASONING TYPE

6.34 **Statement-1 :** If A = {1, 2, 3} & B = {2, 2, 1, 3, 3} are equal.

Statement-2 : A set does not change if one or more elements of the set are repeated.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

6.35 **Statement - 1 :** Total number of relations that can be defined from set A = {1, 2, 3} to a set B = {a, b} is 64

Statement - 2 : If n(A) = p and n(B) = q then total number of relations from A to B is 2^{pq}

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

6.36 **Statement - 1 :** Let A = {1, 2, 3} and R = {(1, 1), (2, 3)}, then R is reflexive relation on A.

Statement - 2 : A relation R on a set A is said to be reflexive if every elements of A is related to itself.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

TOPIC 7 FUNCTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

7.1 The integral points of domain of the function $f(x) = {}^{(2x^2-7x+11)}C_{(7x-x^2-6)} + \log_{[x+1/3]}|x^2-2x-3|$ where [.] denotes the greatest integer function are -

(1) {2, 4, 5, 6}
(2) $\left[\frac{5}{3}, \infty\right) - \{2, 3, 4, 5, 6\}$
(3) $\left[\frac{5}{3}, \infty\right) - \{3\}$
(4) {2, 3, 4, 5, 6}

7.2 If $f(x) = \cos^{-1}\left(\frac{\sqrt{2x^2+1}}{x^2+1}\right)$, then range of f(x) is

(1) $[0, \pi]$
(2) $\left(0, \frac{\pi}{4}\right]$
(3) $\left(0, \frac{\pi}{3}\right]$
(4) $\left[0, \frac{\pi}{2}\right)$

7.3 If f(x) is defined on (0, 1), then the domain of $f(e^x) + f(\ell n\,|x|)$ is -

(1) (–e , –1)
(2) (–e , –1) ∪ (1, e)
(3) (–∞ , –1)
(4) (–e , e)

7.4 The set of possible values of a for which the function $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is one one is

(1) [1, 4]
(2) (1, 4)
(3) (–∞ , 1)
(4) (4, ∞)

7.5 The domain of the function $f(x) = \sqrt{\sin^{-1}(\log_3 x)} + \frac{\tan^{-1} x}{\sqrt{x^2-5x+6}}$ is

(1) [1, 3]
(2) [1, 3)
(3) [1, 2) ∪ (2, 3]
(4) [1, 2)

7.6 If $F(x) = \log(\text{cosec}^{-1}x) + \frac{x^2-3x+2}{x^2-2x+1} + \sqrt{4[x]-[x]^2}$, where [.] = greatest integer function, then the largest interval of x for which F(x) is defined, is

7.7 The range of the function $f(x) = |x-1| + |x-2|,\ -1 \le x \le 3$ is

(1) [1, 3]
(2) [1, 5]
(3) [3, 5]
(4) [1, 5)

7.8 The range of the function $f(x) = \ell n(\sin^{-1}(x^2+x))$ is

(1) $\left[-\ell n\left(\sin^{-1}\frac{1}{4}\right), \ell n\frac{\pi}{4}\right]$
(2) $\left[-\ell n\frac{\pi}{2}, \ell n\frac{\pi}{2}\right]$
(3) $\left(0, \ell n\frac{\pi}{2}\right]$
(4) $\left(-\infty, \ell n\frac{\pi}{2}\right]$

7.9 The domain of function $f(x) = \frac{\sec^{-1} x}{\sqrt{x-[x]}}$, where [.] is greatest integer function.

(1) $R - \{(-1, 1) \cup (n, n \in I)\}$
(2) $R - (-1, 1)$
(3) $R^+ - (0, 1)$
(4) $R - (n, n \in I)$

6.30 Let a relation R be defined by R = {(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)}, then R^{-1} oR is

(1) Reflexive only (2) Symmetric only (3) Transitive only (4) Equivalence

6.31 Let R be a relation over the set N × N and it is defined by (a, b)R(c, d) ⇒ a + d = b + c. Then R is NOT

(1) Reflexive (2) Symmetric (3) Transitive (4) Anti Symmetric

6.32 Let R_1 be a relation defined by $R_1 = \{(a, b) \mid a \geq b,\ a, b \in R\}$. Then R_1 is

(1) Only reflexive (2) Both Reflexive and transitive

(3) Symmetric, transitive but not reflexive (4) Neither transitive nor reflexive but symmetric

6.33 Let A = {1, 2, 3, 4} and R be a relation in A given by R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1), (3, 1), (1, 3)}. Then R is

(1) Reflexive (2) Anti symmetric (3) Transitive (4) An equivalence relation

SECTION - II : ASSERTION & REASONING TYPE

6.34 **Statement-1 :** If A = {1, 2, 3} & B = {2, 2, 1, 3, 3} are equal.

Statement-2 : A set does not change if one or more elements of the set are repeated.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

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Statement - 2 : A relation R on a set A is said to be reflexive if every elements of A is related to itself.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

7.18 Let $f(x) = x^{105} + x^{53} + x^{27} + x^{13} + x^3 + 3x + 1$. If g(x) is inverse of function f(x), then the value of $g'(1)$ is

(1) 3 (2) $\frac{1}{3}$ (3) $-\frac{1}{3}$ (4) not defined

7.19 The function f(x) is defined for all real x, if $f(x + y) = f\left(\frac{xy}{4}\right)$ $\forall$ x, y and $f(-4) = -4$, then f(2011) is

(1) 2010 (2) 2012 (3) 4 (4) – 4

7.20 If $2f(xy) = (f(x))^y + (f(y))^x$ for all x, y $\in$ R and f(1) = 3, then the value of $\sum_{r=1}^{10} f(r)$

(1) $\frac{3}{2}(3^{10} - 1)$ (2) $\frac{3}{2}(3^9 - 1)$ (3) $\frac{3^{10}-1}{2}$ (4) None of these

7.21 The range of the function $f(x) = 5\cos x + 3\cos\left(x + \frac{\pi}{3}\right) + 4$ is

(1) [–3, 11] (2) [–18, 10] (3) [–10, 18] (4) None of these

Level : II (Tough)

7.22 If $g(x) = \log_{f^2(x)}\left(\frac{f(x)-1}{f(x)-2}\right) + (3f(x) - 3)^{2/3} + \sin^{-1}\left(\frac{f(x)}{7}\right) + \sqrt{\cos(\sin f(x))}$, where f(x) is a real valued function, then the range of f(x) for which g(x) is defined.

(1) [–7, 7] (2) $[-7, -1) \cup (-1, 1) \cup (2, 7]$
(3) $[-7, 1) \cup (2, 7]$ (4) None of these

7.23 If f(x) is a polynomial satisfying $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and f(3) = 82, then f(2) is equal to :-.

(1) 16 (2) 17 (3) 19 (4) 21

7.24 If solution of the inequality $x > \sqrt{1-x}$ is (a, b) then the value of 2a + b is :-

(1) $\sqrt{5}$ (2) $-\sqrt{5}$ (3) $2\sqrt{5}$ (4) $-2\sqrt{5}$

7.25 The range of function $f(x) = 3|\sin x| - 4|\cos x|$ is :-

(1) (–4, 3) (2) [–4, 3] (3) (–3, 4) (4) [–3, 4]

7.26 The fundamental period of the function $f(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3x + \sin 3\pi x - 1$ is :-

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) None of these

7.27 Let function $f : X \to Y$, defined as $f(x) = x^2 - 4x + 5$ is invertible and its inverse is $f^{-1}(x)$, then

(1) $X = [2, \infty)$, $Y = [1, \infty)$, $f^{-1}(x) = 2 + \sqrt{x-1}$ (2) $X = (-\infty, 2]$, $Y = [1, \infty)$, $f^{-1}(x) = 2 + \sqrt{x-1}$
(3) $X = (-\infty, \infty)$, $Y = [1, \infty)$, $f^{-1}(x) = 2 - \sqrt{x^2+1}$ (4) none of these

7.28 $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + ax + b$ $\forall x \in R$, then find least value of 'a' for which f(x) is injective function

(1) $\frac{1}{4}$ (2) 1 (3) $\frac{1}{2}$ (4) $\frac{1}{8}$

SECTION - II : ASSERTION & REASONING TYPE

7.29 **Statement-1 :** If $f(x) = \log \sqrt{\frac{1-x}{x}}$, $g(x) = [x]$, then $f(g(x))$ is not defined

Statement-2 : For fog be defined, Range of $g(x) \cap$ Domain of $f(x) \neq \phi$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

7.30 **Statement-1 :** $f(x) = \sin x + \cos x$, defined for R^+ cannot be a periodic function.

Statement-2 : Domain of a periodic function should be unbounded.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False.

(4) Statement-1 is False, Statement-2 is True.

TOPIC 8 LIMIT OF FUNCTION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

8.1 If $f(x) = \begin{cases} 3x-1 & , x \geq 1 \\ 2x+3 & , x < 1 \end{cases}$, $g(x) = \begin{cases} 3-x & , x < 2 \\ 2x-3 & , x \geq 2 \end{cases}$, then $\lim_{x \to 2} f(g(x)) =$

(1) 7 (2) 2 (3) 4 (4) 5

8.2 $\lim_{x \to 0} \frac{e^{[|\sin x|]}}{[x+1]}$ is , where [.] deonotes the greatest integer function.

(1) 1 (2) π (3) does not exit (4) None of these

8.3 $\lim_{x \to \infty} \frac{\cot^{-1} x}{\operatorname{cosec}^{-1} x} =$

(1) 0 (2) 1 (3) does not exist (4) None of these

8.4 $\lim_{x \to 0^+} \frac{-1+\sqrt{1+4(\tan x - \sin x)}}{-1+\sqrt{1+4x^3}}$ is equal to-

(1) $\frac{1}{2}$ (2) 0 (3) $-\frac{1}{2}$ (4) 2

8.5 Value of $\lim_{x \to 0} \frac{\ell n(1+2x-3x^2+4x^3)}{\ell n(1-5x+6x^2-7x^3)}$ is –

(1) 0 (2) $\frac{3}{4}$ (3) $-\frac{2}{5}$ (4) $-\frac{5}{4}$

8.6 $\lim_{x \to 0} \frac{3^x - 1}{\sqrt{1+x} - 1}$ is equal to–

(1) $\ell n 3$ (2) $2\ell n 3$ (3) $3\ell n 3$ (4) 0

8.7 If $\lim_{x \to -n} \frac{x^7 + n^7}{x+n} = 7$, then the value of n is–

(1) ± 5 (2) 0 (3) ± 2 (4) ± 1

8.8 The value of $\lim_{x \to 2} \frac{\tan(e^{x-2} - 1)}{\ell n(x-1)}$ is–

(1) 2 (2) –2 (3) 1 (4) –1

8.9 The value of $\lim_{x \to 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{11x^2}$ is equal to–

(1) $\frac{5}{24}e$ (2) 0 (3) $\frac{11}{24}e$ (4) $\frac{e}{24}$

8.10 The value of $\lim_{x\to 0} \dfrac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}$ is-

(1) $\frac{2}{53}$ (2) $\frac{7}{64}$ (3) $\frac{3}{71}$ (4) $\frac{5}{7}$

8.11 The value of $\lim_{x\to 0^+} (\sin x)^{1/\ell n x}$ is-

(1) 0 (2) e^{-1} (3) e^2 (4) e

8.12 The value of $\lim_{x\to\infty} \dfrac{(\ell n x)^2}{x^2}$ is–

(1) 0 (2) 1 (3) 2 (4) 3

8.13 $\lim_{x\to\infty} f(x)$, where $\dfrac{2x-3}{x} < f(x) < \dfrac{2x^2+5x}{x^2}$, is–

(1) 1 (2) 2 (3) –1 (4) –2

8.14 Value of $\lim_{x\to 0} \dfrac{\sqrt[3]{1+\tan x}-\sqrt[3]{1-\tan x}}{x}$ is–

(1) $\frac{1}{2}$ (2) $-\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

8.15 Value of $\lim_{x\to\frac{\pi}{2}} \tan x \,.\, \ell n \sin x$ is–

(1) 0 (2) $\frac{1}{2}$ (3) $\frac{3}{4}$ (4) 1

8.16 $\lim_{n\to\infty} \left(2^{1/2}.2^{1/4}.2^{1/8}.......2^{1/2^n}\right)$ equals–

(1) 2^0 (2) 2^1 (3) 2^2 (4) 2^3

8.17 The value of $\lim_{\substack{x\to 1\\ y\to 0}} \dfrac{y^3}{x^3-y^2-1}$ as $(x, y) \to (1, 0)$ along the line $y = x - 1$ is–

(1) 0 (2) 1 (3) 2 (4) –1

8.18 If $\lim_{x\to 0} \dfrac{x(1+a\cos x)-b\sin x}{x^3} = 1$ then value of a + b :

(1) –4 (2) –6 (3) 1 (4) none of these

8.19 The value of $\lim_{n\to\infty} \left(1+\sin\dfrac{a}{n}\right)^n$ is :

(1) 1 (2) e^a (3) e^{2a} (4) e^{-a}

Level : II (Tough)

8.20 If $\lim_{x\to 1}\left(\frac{x^n-1}{n(x-1)}\right)^{\frac{1}{x-1}} = e^p$, then p is equal to-

(1) $\frac{n-1}{2}$ (2) $\frac{n+1}{2}$ (3) $\frac{n+3}{2}$ (4) $\frac{n-3}{2}$

8.21 $\lim_{x\to\infty}\left\{\sqrt{x^4-x^2+1}-ax^2-a\right\}$ = A finite value. Then a is equal to–

(1) –1
(2) 1
(3) a cannot be determined
(4) none of these

8.22 $\lim_{x\to 0}\frac{e^x-e^{x\sec x}}{x+\tan x}$ is equal to

(1) 0 (2) 1 (3) 1/2 (4) none of these

8.23 $\lim_{x\to 1}\frac{(\log(1+x)-\log 2)(3.4^{x-1}-3x)}{\{(7+x)^{1/3}-(1+3x)^{1/2}\}\sin\pi x}$ is equal to

(1) $\frac{\pi}{9}\log\frac{4}{e}$ (2) $\frac{9}{\pi}\log\frac{4}{e}$ (3) $\frac{\pi}{9}\log\frac{e}{4}$ (4) none of these

8.24 Evaluate : $\lim_{x\to 0}\left(\frac{e^{x\ell n(3^x-1)}-(3^x-1)^x\sin x}{e^{x\ell n x}}\right)^{1/x}$ is equal to

(1) $\frac{1}{e}\ell n3$ (2) e ℓn 3 (3) 3 (4) $\frac{1}{3}$

8.25 $\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$ is equal to

(1) 0 (2) 1 (3) – 1 (4) 2

SECTION - II : ASSERTION & REASONING TYPE

8.26 **Statement-1 :** If the graph of the function y = f(x) has a unique tangent at the point (a, 0) through which the graph passes then $\lim_{x\to a}\frac{\log_e(1+6(f(x))}{3f(x)} = 2$.

Statement-2 : Since the graph passes through (a, 0). Therefore f(a) = 0. When f(a) = 0 given limit is zero by zero form so that it can be evaluate by using L-Hospital's rule.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

8.27 **Statement-1 :** $\lim_{x\to 0^+} x \sin \frac{1}{x} = 1$

Statement-2 : $\lim_{y\to \infty} y \sin \frac{1}{y} = 1$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

8.28 **Statement-1 :** $\lim_{x\to 0} \left(\frac{g(2-x^2)}{g(2)}\right)^{\frac{4}{x^2}} = e^{-\frac{1}{2}}$, where g(2) = –40 & g′(2) = –5.

Statement-2 : If $\lim_{x\to a} f(x) = 1$, $\lim_{x\to a} g(x) \to \infty$ then $\lim_{x\to a} \{f(x)\}^{g(x)} = e^{\lim_{x\to a} (f(x)-1)g(x)}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

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TOPIC 9 CONTINUITY & DERIVABILITY

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

9.1 If $f(x) = a\,|\sin^7 x| + be^{|x|} + c\,|x|^5$ and if f(x) is differentiable at x = 0 then which of the following is necessarily true -
(1) a = b = c (2) a = 0 , b = 0 , c ∈ R (3) b = c = 0 , a ∈ R (4) b = 0 and a and c ∈ R

9.2 If $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{x-2}{x+3}$, then number of points of discontinuity in f(g(x)) is
(1) 1 (2) 2 (3) 3 (4) 4

9.3 If $f(x) = \begin{cases} \frac{|x-1|}{1-x} + a; & x > 1 \\ a+b; & x = 1 \\ \frac{|x-1|}{1-x} + b; & x < 1 \end{cases}$, is continuous at x = 1 then a and b are respectively–
(1) 1, 1 (2) 1, –1 (3) 2, 3 (4) none of these

9.4 If function

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x; & 0 \le x < \pi/4 \\ 2x\cot x + b; & \pi/4 \le x \le \pi/2 \\ a\cos 2x - b\sin x; & \pi/2 < x \le \pi \end{cases}$$, is continuous in [0, π] then a and b are respectively–

(1) π/6, –π/12 (2) –π/6, π/4 (3) –π/3, π/12 (4) π/3, –π/4

9.5 The function $f(x) = \sin(\log_e |x|)$, $x \neq 0$, and 1 if x = 0
(1) is continuous at x = 0 (2) has removable discontinuity at x = 0
(3) has jump discontinuity at x = 0 (4) has oscillating discontinuity at x = 0

9.6 If $f(x) = \begin{cases} 3^x, & -1 \le x \le 1 \\ 4-x, & 1 < x < 4 \end{cases}$, then at x = 1, f(x) will be :
(1) continuous but not differentiable (2) neither continuous nor differentiable
(3) continuous and differentiable (4) differentiable but not continuous

9.7 The function $f(x) = ||x| - 2|$ is not derivable at
(1) exactly one point (2) exactly two point (3) exactly three point (4) exactly four point

9.8 The function $f(x) = \frac{x}{1+|x|}$ is differentiable in :
(1) (–∞, ∞) (2) (–∞, 0) (3) (–∞, 0) ∪ (0, ∞) (4) (0, ∞)

9.9 If $f(x) = |x+1|\{|x| + |x-1|\}$, then number of points of in [–2, 2] where f(x) is not differentiable is–
(1) 1 (2) 2 (3) 3 (4) 4

9.10 Which of the following function are discontinuous in $(0, \pi)$.

(1) $\cos x + \sin x$

(2) $\cos^2 x + \sin^2 x$

(3) $f(x) = \begin{cases} 1, & 0 \le x \le \frac{3\pi}{4} \\ \frac{2}{\sqrt{3}}\sin\left(\frac{4x}{9}\right), & \frac{3\pi}{4} < x < \pi \end{cases}$

(4) $f(x) = \begin{cases} x\sin x, & 0 \le x \le \frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$

9.11 Which of the following is discontinuous function

(1) $|x|$
(2) $x + |x|$
(3) $x\,|x|$
(4) $[x]$

9.12 The function $y = f(x)$ is defined by $x = 2t - |t|$, $y = t^2 + |t|$, $t \in R$ in the interval $x \in [-1, 1]$, then

(1) f(x) is discontinuous at some points
(2) f(x) is differentiable everywhere
(3) f(x) is continuous but not derivable at $x = 0$
(4) f(x) is constant function

9.13 If $f(x) = \begin{cases} (1+|\sin x|)^{a/|\sin x|}, & -\pi/6 < x < 0 \\ b, & x = 0 \\ e^{\tan 2x/\tan 3x}, & 0 < x < \pi/6 \end{cases}$, is continuous at $x = 0$. Then find the value of $\frac{a}{b}$

(1) $\frac{2}{3}e^{-2/3}$
(2) $\frac{2}{3}e^{2/3}$
(3) $\frac{3}{2}e^{2/3}$
(4) $\frac{3}{2}e^{-2/3}$

9.14 A function f(x) is defined as below $f(x) = \frac{\cos(\sin x) - \cos x}{x^2}$, $x \ne 0$ and $f(0) = a$

f(x) is continuous at $x = 0$ if 'a' equals

(1) 0
(2) 4
(3) 5
(4) 6

9.15 If $f(x) = \begin{cases} \tan^{-1}(\tan x); & x \le \frac{\pi}{4} \\ \pi[x] + 1; & x > \frac{\pi}{4} \end{cases}$, then jump of discontinuity is

(where [.] denotes greatest integer function)

(1) $\frac{\pi}{4} - 1$
(2) $\frac{\pi}{4} + 1$
(3) $1 - \frac{\pi}{4}$
(4) $-1 - \frac{\pi}{4}$

9.16 $f(x) = \begin{cases} \frac{\sqrt{(1+px)} - \sqrt{(1-px)}}{x}, & -1 \le x < 0 \\ \frac{2x+1}{x-2}, & 0 \le x \le 1 \end{cases}$ is continuous in the interval $[-1, 1]$, then 'p' is equal to:

(1) -1
(2) $-1/2$
(3) $1/2$
(4) 1

9.17 If $f(x) = \begin{cases} \frac{x(3e^{1/x} + 4)}{2 - e^{1/x}}, & x \ne 0 \\ 0, & x = 0 \end{cases}$, then f(x) is

(1) continuous as well differentiable at $x = 0$
(2) continuous but not differentiable at $x = 0$
(3) neither differentiable at $x = 0$ nor continuous at $x = 0$
(4) none of these

9.18 The function $f(x) = \sin^{-1}(\cos x)$ is:

(1) discontinuous at $x = 0$
(2) continuous at $x = 0$
(3) differentiable at $x = 0$
(4) none of these

Level : II (Tough)

9.19 Let f : R → R be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, f(0) = 0 and f′ (0) = 3 then f(x) is :

(1) 3x (2) 3x + 1 (3) $4x^2$ (4) $4x^2 + 1$

9.20 If $f(x) = \begin{cases} \frac{e^{[x]+|x|}-2}{[x]+|x|} & , x \neq 0 \\ -1 & , x = 0 \end{cases}$, (where [.] denotes G.I.F.) then

(1) f(x) is continuous at x = 0 (2) $\lim_{x\to 0^+} f(x) = -1$

(3) $\lim_{x\to 0^-} f(x) = 1$ (4) None of these

9.21 Let $f(x) = \left|\left(x+\frac{1}{2}\right)[x]\right|$, when $-2 \le x \le 2$. where [.] represents greatest integer function. Then

(1) f(x) is continuous at x = 2 (2) f(x) is continuous at x = 1

(3) f(x) is continuous at x = – 1 (4) f(x) is discontinuous at x = 0

9.22 If $f(x) = [x^2] + \sqrt{\{x\}^2}$, where [.] and {.} denote the greatest integer and fractional part functions respectively, then-

(1) f(x) is continuous at all integral points (2) f(x) is continuous and differentiable at x = 0

(3) f(x) is discontinuous for all $x \in I - \{1\}$ (4) f(x) is not differentiable for all $x \in I$.

9.23 The value of $\lim_{x\to\pi} \frac{1}{(x-\pi)}\left(\sqrt{\frac{4\cos^2 x}{2+\cos x}} - 2\right)$ is

(1) 0 (2) 2 (3) – 2 (4) does not exist

9.24 Let f(x) = sin x

$g(x) = \begin{cases} \{\max.f(t), 0 \le t \le x\} & \text{for } 0 \le x \le \pi \\ \frac{1-\cos x}{2} & \text{for } x > \pi \end{cases}$

Then number of points in (0, ∞) where f(x) is not differentiable is–

(1) 0 (2) 1 (3) 2 (4) 3

9.25 A function f : R → R satisfies the equation f(x + y) = f(x) f(y) for all x, y ∈ R and f(x) ≠ 0 for any x in R. Let function be differentiable at x = 0 and f ′(0) = 2. Then

(1) f(x) is discontinuous at x = 0

(2) $f(x) = e^{2x}$

(3) f(x) is not differentiable at infinitely many points

(4) none of these

9.26 $g(t) = \lim_{x\to 0} (1 + a \tan x)^{t/x}$, a is even prime number, then find g(2)

(1) e^2 (2) e^3 (3) e^4 (4) none of these

SECTION - II : ASSERTION & REASONING TYPE

9.27 **Statement - 1 :** The function $f(x) = \{x\}$, where $\{.\}$ denotes the fractional part function is discontinuous at $x = 1$

Statement - 2 : $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

9.28 **Statement-1 :** Let $f(x) = [\cos x + \sin x]$, $0 < x < 2\pi$, where $[x]$ denotes the integral part of x then f(x) is discontinuous at 5 points.

Statement-2 : For $x = \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}$, right hand limit not equal to left hand limit.

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

9.29 **Statement-1 :** If $|f(x)| \leq |x|$ for all $x \in R$ then $|f(x)|$ is continuous at 0.

Statement-2 : If f(x) is continuous then $|f(x)|$ is also continuous.

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

9.30 **Statement-1 :** If $f(x + y) = f(x) + f(y)$, then f is either differentiable everywhere or not differentiable everywhere.

Statement-2 : Any function is either differentiable everywhere or not differentiable everywhere

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

TOPIC 10 METHOD OF DIFFERENTIATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

10.1 If $y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ $x \in (0,\pi)$, then $\frac{dy}{dx}$ is equal to

(1) sec x (2) cosec x (3) 1 (4) $-\frac{1}{2}$

10.2 If $f(x) = \frac{x\sqrt{2x+1}}{2x-1}$, then $f'(0)$ is equal to

(1) 1 (2) –1 (3) 0 (4) 2

10.3 If $x = \frac{\sin t}{2+\cos t}$ & $y = \frac{2\cos t}{1+\cos t}$, then $\frac{dy}{dx}$ at $t = \frac{\pi}{2}$ is

(1) 0 (2) – 2 (3) –8 (4) 1

10.4 If $x^y . y^x = e^{x^2}$ then $\frac{dy}{dx}$ at x = 1 is equal to

(1) e(1 – e) (2) $e^2 - e$ (3) $\frac{1-e}{1+e}$ (4) $\frac{1+e}{1-e}$

10.5 If $f(x) = x^n$ then

$f(1) + \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \ldots\ldots + \frac{f^n(1)}{n!}$ is equal to (Here $f^n(x)$ is n^{th} derivative of f(x) w.r.t. x)

(1) n (2) 2^n (3) 2^{n-1} (4) $\frac{n(n+1)}{2}$

10.6 If $(\sin x)^y = (\cos y)^x$ then $\frac{dy}{dx}$ equals

(1) $\frac{\ell n\cos y - y\tan x}{\ell n\sin x + x\tan y}$ (2) $\frac{\ell n\sin x + x\tan y}{\ell n\cos y - y\cot x}$ (3) $\frac{\ell n\cos y - y\tan x}{\ell n\sin x - x\tan y}$ (4) $\frac{\ell n\cos y - y\cot x}{\ell n\sin x + x\tan y}$

10.7 If $y = \sin x + e^x$, then $\frac{d^2x}{dy^2}$ is equal to

(1) $\frac{\sin x - e^x}{(\cos x + e^x)^3}$ (2) $\frac{\sin x - 2e^x}{(\cos x + e^x)^3}$ (3) $\frac{2\sin x - y}{(\cos x - \sin x)^3}$ (4) $(\sin x - e^x)\left(\frac{dy}{dx}\right)^3$

10.8 S_1 : If $f(x) = |x^2 - 5|x| + 6|$ then $f'\left(\frac{-5}{2}\right) = 0$

S_2 : The slope of tangent at x = 1 on the curve $y = \sin^{-1}(\cos \pi x)$ is π

S_3 : If x = f(t), y = g(t) then $\frac{d^2x}{dy^2} = \frac{f''(t)}{g''(t)}$

(1) TFT (2) TFF (3) FFT (4) FFF

10.9 S_1 : If $x^3 + y^2 = 5(xy-1)$ then $\frac{dy}{dx}$ at (1, 3) is equal to 12

S_2 : If $x^e = e^y$ then $\frac{dy}{dx}$ at x = 1 is equal to e

S_3 : If f(x) is a linear polynomial then $f'(\sin x) + f''(\sin x)$ is constant

(1) TFT (2) TFF (3) TTT (4) FFF

10.10 If f(xy) = f(x).f(y) and f(3) = 1, then f′(10) is equal to ....

(1) 10 (2) 1 (3) 0 (4) none of these

10.11 If x = cost, y = ℓnt, then find the value of $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$ at $t = \frac{\pi}{2}$

(1) 1 (2) 0 (3) $\pi/2$ (4) none of these

10.12 If y = |cos x| + |sin x|, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is

(1) $\frac{\sqrt{3}+1}{2}$ (2) $2(\sqrt{3} - 1)$ (3) $\frac{1}{2}(\sqrt{3} - 1)$ (4) None of these

10.13 If $y = f\left(\frac{3x+4}{5x+6}\right)$ and $f'(x) = \tan x^2$, then $\frac{dy}{dx}$ is equal to

(1) $-2 \tan \left(\frac{3x+4}{5x+6}\right)^2 \cdot \frac{1}{(5x+6)^2}$ (2) $f\left(\frac{3\tan x^2+3}{5\tan x^2+6}\right) \tan x^2$

(3) $2x \tan \left(\frac{3x+4}{5x+6}\right)$ (4) $\tan x^2$

10.14 If $y = \tan^{-1}\left(\frac{2^x}{1+2^{2x+1}}\right)$, then $\frac{dy}{dx}$ at x = 0 is

(1) 1 (2) 2 (3) $\frac{-3}{5}\log_e 2$ (4) $\frac{-\log_e 2}{10}$

10.15 If $y = \frac{1}{1+x^{n-m}+x^{p-m}} + \frac{1}{1+x^{m-n}+x^{p-n}} + \frac{1}{1+x^{m-p}+x^{n-p}}$, then $\frac{dy}{dx}$ is equal to

(1) 1 (2) 0 (3) m + n + p (4) m − n + p

Level : II (Tough)

10.16 If $e^{x+y} = x^y$ then

(1) $\frac{dy}{dx} = 0$ at x = 1 (2) $\frac{dy}{dx}$ does not exist at x = e

(3) $\frac{dy}{dx} = e$ at x = e (4) none of these

10.17 If $y = 2\sin^{-1}\sqrt{1-x} + \sin^{-1}2\sqrt{x(1-x)}$, $x \in \left(0, \frac{1}{2}\right)$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{1}{\sqrt{1-x^2}}$ (2) $-\frac{1}{\sqrt{1-x^2}}$ (3) 0 (4) none of these

10.18 If f be a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x) = (f(x))^2 + (g(x))^2$, then find the value of h(10), if h(0) = 1

(1) 1 (2) –1/2 (3) 10 (4) can not be determined

10.19 If $y = x^{\ell nx} . \tan^{-1}x$, then find the value of $\frac{dy}{dx}$ at x = 1

(1) 1 (2) $\pi/2$ (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

10.20 If $y = (e^x)^{(e^x)^{(e^x)^{\cdots\cdots\infty}}}$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{y^2}{1-xy}$ (2) $\frac{e^{2xy}}{1-xy}$ (3) $\frac{(x-1)y}{xy}$ (4) Both (1) and (2) are correct

10.21 The implicit equation $x^2 + 5xy + y^2 - 2x + y - 6 = 0$, then find $\frac{dy}{dx}$ at (1, 1)

(1) $\frac{5}{8}$ (2) $-\frac{5}{8}$ (3) $\frac{8}{5}$ (4) $-\frac{8}{5}$

SECTION - II : ASSERTION & REASONING TYPE

10.22 **Statement-1 :** If $y = (1 + x)(1 + x^2)(1 + x^4)\ldots\ldots(1+x^{2^n})$, then $y'(0) = 1$

Statement-2 : $\frac{d}{dx}(\ell nx) = \frac{1}{x}$ for $x > 0$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

10.23 **Statement - 1** For $x < 0$, $\frac{d}{dx}(\ell n\,|x|) = \frac{1}{x}$.

Statement - 2 For $x < 0$, $|x| = -x \Rightarrow \frac{d}{dx}|x| = -1$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

10.24 **Statement - 1** Let $f : [0, \infty) \to [0, \infty)$ be a function defined by $y = f(x) = x^2$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right) = 1$.

Statement - 2 $\frac{d^2y}{dx^2} = -\frac{d^2x}{dy^2}\cdot\left(\frac{dy}{dx}\right)^3$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 11 APPLICATION OF DERIVATIVE

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

11.1 Let $f(x) = 1 + |x - 2| + |\sin x|$, then lagrange's means value theorem is applicable for f(x) in -

(1) $[0, \pi)$ (2) $[\pi, 2\pi]$ (3) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (4) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

11.2 If $f(x) = a\log|x| + bx^2 + x$ has extremums at $x = 1$ and $x = 3$, then -

(1) $a = -\frac{3}{4}$, $b = \frac{1}{8}$ (2) $a = -\frac{3}{4}$, $b = -\frac{1}{8}$ (3) $a = \frac{3}{4}$, $b = -\frac{1}{8}$ (4) None of these

11.3 Let $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$, $a \in R$ then

(1) f(x) is monotonic decreasing in R for $a \in (1, 4)$
(2) f(x) is monotonic increasing in R for $a \in (1, 4)$
(3) f(x) is not invertible for $a \in [1, 4]$
(4) f(x) is not invertible for $a \in (1, 4)$

11.4 Find the number of all the possible integral values of λ for which the curve $y = \frac{x^4}{4} - \frac{3x^2}{2} + \lambda x - 3$ has three tangents parallel to the axis of x.

(1) 1 (2) 2 (3) 3 (4) 4

11.5 Let $f(x) = \begin{cases} x^{\alpha}\sin\frac{\pi}{nx} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, where $n \in I$, $n \neq 0$. If Rolle's theorem is applicable to f(x) in the interval [0, 1], then

(1) for $\alpha > 0$, least value of n is -2
(2) for $\alpha > 1$, greatest value of n is -1
(3) for $\alpha > 0$, greatest value of n is 1
(4) for $\alpha < 0$, least value of n is -1

11.6 The points of contact of the vertical tangents to $x = 2 - 3\sin\phi$, $y = 3 + 2\cos\phi$ are :

(1) (2, 5), (2, 1) (2) (3, 5), (3, –1) (3) (–1, 3), (5, 3) (4) (–1, 3), (2, 1)

11.7 The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that :

(1) it makes constant angle with x-axis
(2) it passes through the origin.
(3) it is at a constant distance from (0, 0)
(4) none of these

11.8 If the line $Ax + By + C = 0$ is normal to the curve $xy = 1$. then

(1) $A > 0, B > 0$ (2) $A > 0, B < 0$ (3) $A < 0, C > 0$ (4) $A < 0, B < 0$

11.9 Consider curve $y = f(x)$, then

if PT = Length of tangent
PN = Length of normal
TM = Length of subtangent
MN = Length of sub normal, then

(1) TM . MN = $f'(x)$ (2) $\frac{PT}{PN} = \frac{1}{f'(x)}$ (3) PT . PN = constant (4) PT . TM = constant

11.10 If law of linear motion of a particle is given by $S = \frac{1}{3}t^3 - 16t$, then the acceleration at the time when the velocity vanishes, is

(1) 0 (2) 2 (3) 4 (4) 8

11.11 Let f(x) satisfies the requirement of lagrange mean value theorem in (0, 2). If f(0) = 0 and $f'(x) \le \frac{1}{2}$, $x \in [0, 2]$, then

(1) $|f(x)| \le 2$ (2) $f(x) \le 1$
(3) $f(x) = 2x$ (4) $f(x) = 3$ for at least one x in [0, 2]

11.12 If the function $f(x) = ax^3 + bx^2 + 11x - 6$ satisfies conditions of Rolle's theorem in [1, 3] and $f'\left(2+\frac{1}{\sqrt{3}}\right) = 0$, then values of a and b are respectively.

(1) 1, – 6 (2) – 2, 1 (3) $-1, \frac{1}{2}$ (4) –1, 6

11.13 From mean values theorem :

$f(b) - f(a) = (b - a) f'(x_1)$; $a < x_1 < b$ if $f(x) = \frac{1}{x}$, x_1 is equal to

(1) $\sqrt{ab}$ (2) $\frac{a+b}{2}$ (3) $\frac{2ab}{a+b}$ (4) ab (b – a)

11.14 For x > 1, $y = \ell n\, x + 1 - x$ satisfies the inequality

(1) $x^2 - 1 > \ell n\, x$ (2) $x - 1 > \ell n\, x$ (3) $x - 1 < \ell n\, x$ (4) $\ell n\, x > x$

11.15 If $f(x) = \begin{cases} -x & , \quad 2 < x \le 3 \\ x^2 + 12x - 1 & , \quad -1 \le x \le 2 \end{cases}$, then

(1) f(x) is increasing on [–1, 2] (2) f(x) is decreasing on [–1, 2]
(3) f′(x) at x = 2, does not exist (4) f(x) has minimum at x = 2

11.16 Which of the following does not holds for $y = e^x$

(1) $e^2 + \frac{1}{e} > 2\sqrt{e}$ (2) $e^1 + e^{-2} + e^3 > e^{2/3}$ (3) $e^{x_1} + e^2 > 2e^{\left(\frac{x_1+2}{2}\right)}$, $x_1 \in R$ (4) none of these

11.17 The values of a for which $f(x) = \frac{a^2x^3}{3} + \frac{3ax^2}{2} + 2x + 1$ is strictly decreasing at x = 1

(1) $a \in (-2, -1)$ (2) $a \in (-1, 0)$ (3) $a \in (1, 2)$ (4) $a \in (-2, 1)$

11.18 For $f(x) = \sin^2 x$, $x \in (0, \pi)$ point of inflection is :

(1) $\frac{\pi}{6}$ (2) $\frac{2\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $\frac{4\pi}{3}$

11.19 The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5\, dt$ has a local maximum at x =

(1) 0 (2) 1 (3) 2 (4) 3

11.20 Let (h, k) be a fixed point, where $h > 0, k > 0$. A straight line passing through this point cut the positive direction of the co-ordinate axes at the points P and Q. Which of the following is the minimum area of triangle OPQ, O being the origin :

(1) hk (2) 2hk (3) $\frac{1}{2}$ hk (4) h^2k^2

11.21 Let $f(x) = \begin{cases} x^2 & , x < -1 \\ -x & , -1 \le x \le 0 \\ kx^2 + p & , x > 0 \end{cases}$, then for what value of (k, p), f(x) has a minima

(1) (2, –1) (2) (6, 0) (3) (–2, 0) (4) (0, – 6)

11.22 A right circular cylinder of maximum volume is inscribed in a given right circular cone of height h and base radius r, then radius of cylinder is :

(1) $\frac{3r}{2}$ (2) $\frac{2r}{3}$ (3) $\frac{r}{3}$ (4) $\frac{r}{2}$

11.23 Values of n for which the length of subnormal of the curve $xy^n = a^{n+1}$ is constant

(1) 3 (2) – 2 (3) – 3 (4) $\frac{1}{2}$

11.24 The values of λ for which the function $f(x) = \lambda x^3 - 2\lambda x^2 + (\lambda + 1)x + 3\lambda$ is increasing through out number line.

(1) $\lambda \in (-3, 3)$ (2) $\lambda \in (-3, 0)$ (3) $\lambda \in (0, 3)$ (4) $\lambda \in (1, 3)$

11.25 If $0 < x_1 < x_2$ which of following is true for $y = \sec^{-1}x$

(1) $\sec^{-1} x_1 + \sec^{-1}x_2 > \sec^{-1}\left(\frac{x_1 + x_2}{2}\right)$ (2) $\sec^{-1} x_1 + \sec^{-1} x_2 < 2\sec^{-1}\left(\frac{x_1 + x_2}{2}\right)$

(3) $\sec^{-1} x_1 > \sec^{-1} x_2$ (4) $\sec^{-1} x_1 = \sec^{-1} x_2$

11.26 Let f be differentiable $(x \in R)$. if $f(1) = -2$ and $f'(x) \ge 2$ for $x \in [1, 6]$, then

(1) f(6) < 6 (2) f(6) ≥ 8 (3) f(6) = 5 (4) f(6) ≤ 5

11.27 Let $f(x) = \frac{x^2 + 2}{[x]}$, $1 \le x \le 3$, where [.] represents greatest integer function, then

(1) f(x) is increasing in [1, 3] (2) least value of f(x) is 3

(3) greatest value of f(x) is $\frac{11}{2}$ (3) f(x) has no greatest value

11.28 Shortest distance between $|x| + |y| = 2$ and $x^2 + y^2 = 16$ is -

(1) 3 (2) 2 (3) 1 (4) 5

11.29 The distance between two horizontal tangents of $y = x^3 + x^2 - x$ is -

(1) $\frac{27}{32}$ (2) $\frac{32}{27}$ (3) $\frac{2}{3}$ (4) $\frac{1}{32}$

11.30 Number of solution of equation $\sin x = x^2 + x + 1$ is -

(1) 0 (2) 1 (3) 2 (4) 3

Level : II (Tough)

11.31 A cylindrical gas container is closed at the top and open at the bottom, if the iron plate of top is $\frac{5}{4}$ times as thick as the plate forming the cylindrical sides. The ratio of the radius to the height of the cylinder using minimum material for the same capacity is :

(1) $\frac{2}{3}$ (2) $\frac{1}{2}$ (3) $\frac{4}{5}$ (4) $\frac{1}{3}$

11.32 A boat is to be driven 300 km at a constant speed of x kmph. Speed rules required $25 \le x \le 60$. The fuel cost Rs. 10 per litre and is consumed at the rate of $2 + \frac{x^2}{600}$ liters/hour the wages of the driver is Rs. 200/hour. The most economical speed to drive the boat in kmph, is

(1) 50 (2) $50\sqrt{3}$ (3) $20\sqrt{3}$ (4) 60

11.33 Let $f(x) = a_0 + a_1x^2 + a_2x^4 + a_3x^6 + ... + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 .... < a_n$. The function f(x) has :

(1) neither a maxima nor a minima
(2) only one maxima
(3) both maxima and minima
(4) only one minima

11.34 Shortest distance of curve $2x^2 + 5xy + 2y^2 = 1$ from origin is

(1) $\frac{2}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{2}$ (4) $\frac{\sqrt{2}}{3}$

11.35 Let $f(x) = \frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)}$, $a < b < c$. Which of the following are correct ?

(1) f(x) is quadratic expression hence has exactly two zeroes
(2) $\frac{d}{dx}$ f(x) = 0, when $x = \frac{a+b+c}{3}$
(3) $\tan^{-1}(f(a)) < \tan^{-1}(f(b)) < \tan^{-1}(f(c))$
(4) none of these

11.36 $f'(x) \le 2$ and f differentiable for $x \in R$. If f(1) = 2 and f(4) = 8, then f(2) has the value equal to -

(1) 4 (2) 2 (3) 1 (4) 5

11.37 Number of different points on the curve $y = x^4$, where the tangents drawn from the point $\left(\frac{3}{4}, 0\right)$ meet the curve is -

(1) 3 (2) 1 (3) 2 (4) None of these

11.38 The number of critical points of the function $f(x) = x^{\frac{1}{3}}.(x-1)^{\frac{2}{3}}$ is

(1) 0 (2) 1 (3) 2 (4) 3

SECTION - II : ASSERTION & REASONING TYPE

11.39 **Statement 1 :** Let $f(x) = 2 . \tan^{-1}\frac{1-x}{1+x}$ on [0, 1], then the range of $f = \left[0, \frac{\pi}{2}\right]$.

Statement 2 : f decreases from $\frac{\pi}{4}$ to 0.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

11.40 **Statement-1 :** $e^x + e^{-x} > 2 + x^2$, $x \neq 0$
Statement-2 : $f(x) = e^x + e^{-x} - 2 - x^2$ is an increasing function.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

11.41 Let $f(x) = \begin{cases} e^x + 1 & , \quad -1 \le x < 0 \\ e^x & , \quad x = 0 \\ e^x - 1 & , \quad 0 < x \le 1 \end{cases}$

Statement-1 : f is bounded but never reaches its maximum and minimum.
Statement-2 : f is discontinuous at x = 0
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

11.42 **Statement-1 :** If $f(x) = \begin{cases} 2e^x + 1 & x < 0 \\ 3 & x \ge 0 \end{cases}$ then f(x) is strictly increasing.

Statement-2 : f (x) is strictly increasing if $f'(x) \ge 0$, where equality holds for some descrete points.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

11.43 **Statement-1 :** If f(x) is increasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.
Statement-2 : If f(x) is decreasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

11.44 **Statement-1 :** For any triangle ABC $\sin\left(\frac{A+B+C}{3}\right) \ge \frac{\sin A + \sin B + \sin C}{3}$

Statement-2 : y = sinx is concave downward for $x \in (0, \pi)$
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 12 INDEFINITE INTEGRATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

12.1 $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx =$

(1) log|sinx + cosx| + c
(2) $\log|\sin x + \cos x|^2 + c$
(3) log|sinx – cosx| + c
(4) $\frac{-1}{\sin x + \cos x} + c$

12.2 If $\int \sqrt{1+\sec x}\, dx = 2\, fog\,(x) + c$ then -

(1) $g(x) = \sqrt{\tan^{-1} x}$ $f(x) = \sec x - 1$
(2) $g(x) = \sqrt{\sec x - 1}$ $f(x) = 2\tan^{-1} x$
(3) $f(x) = \tan^{-1} x$ $g(x) = \sqrt{\sec x - 1}$
(4) None of these

12.3 $\int \frac{dx}{(4+3x^2)\sqrt{3-4x^2}} =$

(1) $\frac{1}{5}\tan^{-1}\frac{2x}{\sqrt{3-4x^2}} + c$
(2) $\frac{1}{10}\tan^{-1}\frac{5x}{2\sqrt{3-4x^2}} + c$
(3) $\frac{1}{5}\tan^{-1}\frac{5x}{2\sqrt{3-4x^2}} + c$
(4) $\frac{1}{10}\tan^{-1}\frac{5x}{\sqrt{3-4x^2}} + c$

12.4 If $\int g(x)\,dx = g(x)$, then $\int g(x)\,(f(x) + f'(x))\,dx$ is -

(1) $g(x)\, f^2(x) + c$
(2) $g(x)f(x) - g(x)f'(x) + c$
(3) $g(x)\, f'(x) + c$
(4) $g(x)\, f(x) + c$

12.5 $\int \frac{e\cos(e\ln x)dx}{x}$ is equal to

(1) cos (e ln x) + c
(2) sin (e lnx) + c
(3) cos (ln x)
(4) None of these

12.6 $\int \left(3x^2 \tan\frac{1}{x} - x\sec^2\frac{1}{x}\right)dx$ is equal to

(1) $x^3 \cos\frac{1}{x} + c$
(2) $x^2 \tan\frac{1}{x} + c$
(3) $x^3 \tan\frac{1}{x} + c$
(4) $x^2 \sec\frac{1}{x} + c$

12.7 $\int \frac{dx}{x(x^{2010}+1)}$ is equal to

(1) $\frac{1}{2009}\ln|1 + x^{2010}| + c$
(2) $\frac{1}{2010}\ln|1 + x^{-2010}| + c$
(3) $\ln|1 + x^{2010}| + x + c$
(4) $-\frac{1}{2010}\ln|1 + x^{-2010}| + c$

12.8 $\int \sqrt{\frac{x^6}{a^8+x^8}}\,dx =$

(1) $\ln|x^8 + \sqrt{a^8+x^8}| + c$

(2) $\frac{1}{4}\ln|x^4 + \sqrt{a^8+x^8}| + c$

(3) $\frac{1}{8}\ln|x^8 + \sqrt{a^8+x^8}| + c$

(4) $\frac{1}{4}\ln|a^8 + \sqrt{x^8+a^8}| + c$

12.9 $\int \cot^3 x.\operatorname{cosec}^{-8}x\,dx$ is equal to

(1) $\frac{\operatorname{cosec}^8 x}{8} - \frac{\operatorname{cosec}^6 x}{6} + c$

(2) $\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + c$

(3) $\frac{\cos^8 x}{8} - \frac{\sin^8 x}{6} + c$

(4) None of these

12.10 If $\int \tan^7 x\,dx = f(x) + \ln|\cos x| + c$, then

(1) f(x) is a polynomial of degree 8 in tan x

(2) f(x) is a polynomial of degree 5 in tan x

(3) $f(x) = \tan^6 x - \frac{1}{4}\tan^4 x + \frac{1}{2}\tan^2 x + \ln|\cos x| + c$

(4) f(x) is a polynomial of degree 6 in tan x

12.11 $\int \frac{d(\ln x)}{x+1} =$

(1) $\ln\left|\frac{x+1}{x}\right| + c$

(2) $\ln|\ln x + 1| + c$

(3) $\ln\left|\frac{x}{x+1}\right| + c$

(4) $-\ln|1 + \ln x| + c$

12.12 If $\int \frac{x^4+1}{x^6+1}\,dx = A\tan^{-1}\left(x - \frac{1}{x}\right) + B\tan^{-1}x^3 + c$, then

(1) $A + B = \frac{1}{3}$

(2) A is prime number

(3) A is a composite number

(4) B is natural number

12.13 If $\int \frac{dx}{(x^2+1)(x^2+4)} = P\tan^{-1}x + Q\tan^{-1}\frac{x}{2} + c$, then P + Q =

(1) $P + Q = \frac{1}{3}$

(2) $P + Q = \frac{2}{3}$

(3) $P + Q = -\frac{1}{3}$

(4) $P + Q = \frac{1}{6}$

12.14 Evaluate : $\int \frac{(x^2-x+1)}{(x^2+1)^{3/2}} e^x dx$

(1) $\frac{e^x}{\sqrt{1+x^2}} + C$

(2) $\frac{e^x}{\sqrt{1-x^2}} + C$

(3) $\frac{xe^x}{\sqrt{1+x^2}} + C$

(4) $\frac{e^x}{\sqrt{2+x^2}} + C$

12.15 $\int \frac{x^7}{(1-x^2)^5}\,dx$ is equal to :

(1) $\frac{1}{4}\cdot\frac{x^8}{(1-x^2)^4}$

(2) $\frac{1}{8}\cdot\frac{x^4}{(1-x^2)^8}$

(3) $\frac{1}{8}\cdot\frac{x}{(1-x^2)^4}$

(4) $\frac{1}{8}\cdot\frac{x^8}{(1-x^2)^4}$

12.16 If $\int \frac{x^4+1}{x(x^2+1)^2}dx = A\,\ell n\,|x| + \frac{B}{1+x^2} + C$, where C is the constant of integration, then:

(1) A = 1; B = – 1
(2) A = – 1; B = 1
(3) A = 1; B = 1
(4) A = – 1; B = – 1

12.17 The value of $\int x.\frac{\ell n\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}dx$ equals:

(1) $\sqrt{1+x^2}\ \ell n\left(x+\sqrt{1+x^2}\right) - x + C$
(2) $\frac{x}{2}.\ell n^2\left(x+\sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}} + C$
(3) $\frac{x}{2}.\ell n^2\left(x+\sqrt{1+x^2}\right) + \frac{x}{\sqrt{1+x^2}} + C$
(4) $\sqrt{1+x^2}\ \ell n\left(x+\sqrt{1+x^2}\right) + x + C$

12.18 The value of $\int \frac{\ell n|x|}{x\sqrt{1+\ell n|x|}}dx$ equals :

(1) $\frac{2}{3}\sqrt{1+\ell n|x|}\ (\ell n\,|x| - 2) + C$
(2) $\frac{2}{3}\sqrt{1+\ell n|x|}\ (\ell n\,|x| + 2) + C$
(3) $\frac{1}{3}\sqrt{1+\ell n|x|}\ (\ell n\,|x| - 2) + C$
(4) $2\sqrt{1+\ell n|x|}\ (3\,\ell n\,|x| - 2) + C$

12.19 The value of $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x}dx$ is :

(1) $\frac{1}{2}\sin 2x + C$
(2) $-\frac{1}{2}\sin 2x + C$
(3) $-\frac{1}{2}\sin x + C$
(4) $-\sin^2 x + C$

12.20 The value of $\int \frac{x\ dx}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}$ is equal to:

(1) $\frac{1}{2}\ \ell n\left(1+\sqrt{1+x^2}\right) + C$
(2) $2\sqrt{1+\sqrt{1+x^2}} + C$
(3) $2\left(1+\sqrt{1+x^2}\right) + C$
(4) none of these

12.21 The value of $\int \frac{dx}{\sin x.\sin(x+\alpha)}$ is equal to

(1) $\operatorname{cosec}\alpha\ \ell n\left|\frac{\sin x}{\sin(x+\alpha)}\right| + C$
(2) $\operatorname{cosec}\alpha\ \ell n\left|\frac{\sin(x+\alpha)}{\sin x}\right| + C$
(3) $\operatorname{cosec}\alpha\ \ell n\left|\frac{\sec(x+\alpha)}{\sec x}\right| + C$
(4) $\operatorname{cosec}\alpha\ \ell n\left|\frac{\sec x}{\sec(x+\alpha)}\right| + C$

12.22 If $\int \frac{1}{1+\sin x}dx = \tan\left(\frac{x}{2}+a\right) + b$, then

(1) $a = -\frac{\pi}{4}$, $b \in R$ (2) $a = \frac{\pi}{4}$, $b \in R$ (3) $a = \frac{5\pi}{4}$, $b \in R$ (4) none of these

12.23 The value of $\int \frac{\cos 2x}{\cos x}dx$ is equal to

(1) $2\sin x - \ell n|\sec x + \tan x| + C$
(2) $2\sin x - \ell n|\sec x - \tan x| + C$
(3) $2\sin x + \ell n|\sec x + \tan x| + C$
(4) None of these

12.24 The value of $\int 5^{5^{5^x}}.5^{5^x}.5^x\,dx$ is equal to

(1) $\frac{5^{5^x}}{(\ell n 5)^3} + C$ (2) $5^{5^{5^x}}(\ell n\, 5)^3 + C$ (3) $\frac{5^{5^{5^x}}}{(\ell n\, 5)^3} + C$ (4) none of these

Level : II (Tough)

12.25 $\int\left(\log_e(1+\tan x) + \frac{2x}{1+\cos 2x+\sin 2x}\right)dx$ is equal to

(1) $x\,\ell n\,(1+\tan x) + c$ (2) $x^2\,\ell n\,(1+\cos x)$ (3) $x\,\ell n\,(1+\sec x)$ (4) none

12.26 Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}}dx = \frac{2}{3}\,gof(x) + c$ then

(1) $f(x) = \sqrt{x}$, $g(x) = \ln|x - \sqrt{1-x^3}|$
(2) $f(x) = x^{3/2}$, $g(x) = \ln|x - \sqrt{1-x^3}|$
(3) $f(x) = \sqrt{x}$, $g(x) = \sin^{-1}x$
(4) $f(x) = x^{3/2}$, $g(x) = \sin^{-1}x$

12.27 If $\int \frac{x+(\cos^{-1}3x)^2}{\sqrt{1-9x^2}}dx = A\sqrt{1-9x^2} + B(\cos^{-1}(3x))^3 + c$, then

(1) $A - B = \frac{1}{9}$ (2) $A - B = 0$ (3) $A - B = -\frac{1}{9}$ (4) $A - B = -1/2$

12.28 $\int \frac{\sin^3 x\,dx}{(1+\cos^2 x)\sqrt{1+\cos^2 x+\cos^4 x}}$ is equal to :

(1) $\text{cosec}^{-1}(\sec x + \cos x) + C$
(2) $\sec^{-1}(\sec x + \cos x) + C$
(3) $\sec^{-1}(\sec x - \cos x) + C$
(4) $\cos^{-1}(\sec x + \cos x) + C$

12.29 $\int x^{-6}(1+2x^3)^{2/3}dx$ is equal to :

(1) $-\frac{1}{5}(x^{-3}+2)^{5/3} + c$
(2) $\frac{1}{5}(x^{-3}+2)^{5/3} + c$
(3) $-\frac{1}{5}(x^{-3}+2)^{3/5} + c$
(4) $-\frac{1}{5}(x^{-3}-2)^{5/3} + c$

12.30 The value of $2\int \sin x \,.\, \text{cosec}4x \, dx$ is equal to

(1) $\frac{1}{2\sqrt{2}} \ell n \left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| - \frac{1}{4} \ell n \left|\frac{1+\sin x}{1-\sin x}\right| + C$ (2) $\frac{1}{2\sqrt{2}} \ell n \left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + \frac{1}{4} \ell n \left|\frac{1+\sin x}{1-\sin x}\right| + C$

(3) $\frac{1}{2\sqrt{2}} \ell n \left|\frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}\right| - \frac{1}{4} \ell n \left|\frac{1+\sin x}{1-\sin x}\right| + C$ (4) none of these

12.31 $\int \frac{(x-1)^2}{x^4+x^2+1} dx$

(1) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$ (2) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) - \frac{2}{\sqrt{5}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$

(3) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) + \frac{2}{\sqrt{5}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$ (4) $\tan^{-1}\left(\frac{x^2-1}{x\sqrt{3}}\right) + \frac{2}{\sqrt{5}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$

12.32 $\int \frac{1 + x\cos x}{x\left(1 - x^2 e^{2\sin x}\right)} dx$

(1) $\ell n\,(x\, e^{\sin x}) + \frac{1}{2} \ln\,(1 - x^2 e^{2\sin x}) + C$ (2) $\ell n\,(x\, e^{\sin x}) - \frac{1}{2} \ln\,(1 - x^2 e^{2\sin x}) + C$

(3) $\ell n\,(x\, e^{\sin x}) - \frac{3}{2} \ln\,(1 - x^2 e^{2\sin x}) + C$ (4) $\ell n\,(x\, e^{\sin x}) - \frac{3}{2} \ln\,(1 + x^2 e^{2\sin x}) + C$

12.33 The integral of $\int\left(\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right)dx =$

(1) $\ln(\ln x) + c$ (2) $x \ln x + c$ (3) $\frac{x}{\ln x} + c$ (4) None of these

12.34 $\int e^{\sin x}(x\cos x - \sec x \tan x)dx =$

(1) $xe^{\sin x} - e^{\sin x} . \sec x + c$ (2) $(x + \sec x)e^{\sin x}$
(3) $e^{\sin x} \cos x + c$ (4) $e^{\sin x} (\cos x - \sec x) + c$

SECTION - II : ASSERTION & REASONING TYPE

12.35 **Statement-1 :** $\int e^x(\ln \sin x + \cot x)dx = e^x.\ln(\sin x) + c$

Statement-2 : $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

12.36 **Statement-1 :** $\int \tan 3x.\tan 2x.\tan x\,dx = \frac{\ln|\sec 3x|}{3} + \frac{\ln|\sec 2x|}{2} + \ln|\sec x| + c$

Statement-2 : tan 3x . tan 2x . tanx = tan 3x – tan 2x – tan x

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

12.37 **Statement-1 :** $\int \frac{1-\cot^{2010} x\,dx}{\tan x + \cot x.\cot^{2010} x} = \frac{1}{2010}\ell n\left|\sin^{2010} x + \cos^{2010} x\right| + c$

Statement-2 : $\int \frac{dx}{(x+2)^{\frac{2011}{2010}}(x-3)^{\frac{2009}{2010}}} = 402.\left(\frac{x-3}{x+2}\right)^{1/2010} + c$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

12.38 **Statement-1 :** $\int \frac{8^{1+x} + 4^{1-x}}{2^x}dx = \frac{2^{2+2x}}{\ell n\,2} - \frac{2^{2-3x}}{3\ell n 2} + c$

Statement-2 : $\int \frac{3+2\cos x}{(2+3\cos x)^2}dx = \frac{\sin x}{2+3\sin x} + c$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

12.39 **Statement-1 :** $\int (\sin x)^5 \cos x\,dx = \frac{\sin^6 x}{6} + C$

Statement-2 : $\int (f(x))^n f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + C,\ n \in I$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 13 DEFINITE INTEGRATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

13.1 $\int_{1}^{e^{37}} \frac{\pi \sin(\pi \ell n x)}{x} dx =$

(1) 0 (2) 1 (3) 2 (4) –1

13.2 The value of $\int_{-1}^{3} ([x] + |x-2|)\, dx$ is , where [.] denotes GIF

(1) 3 (2) 2 (3) 5 (4) 7

13.3 $\int_{0}^{\pi/2} \log(\tan x)\, dx =$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) 0 (4) 1

13.4 $\int_{0}^{5\pi} [\tan^{-1} x]\, dx$, where [.] denotes the greatest integer function, is equal to

(1) $10\pi - \tan 1$ (2) $5\pi - \tan 1$ (3) $5 - \tan 1$ (4) $10\pi + \tan 1$

13.5 $\int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx =$

(1) $\ell n\, 2$ (2) $2\, \ell n 2$ (3) $\frac{1}{2} \ell n 2$ (4) $\frac{1}{4} \ell n 2$

13.6 $\int_{-\pi/4}^{\pi/4} \frac{x^{11} - 3x^9 + 8x^3 - 4x + 1}{\sec^2 x} dx =$

(1) 0 (2) $\frac{\pi}{2}$ (3) 2 (4) 1

13.7 $\int_{-\pi/4}^{\pi/4} \frac{\sec^2 x}{1 + 3^x} dx =$

(1) 2 (2) 1 (3) 0 (4) 4

13.8 If $f(x) = \sin x + \int_{0}^{\pi/2} \sin x \cos t\, f(t) dt$, then $\int_{0}^{\pi/2} f(x) dx =$

(1) 0 (2) 2 (3) 1 (4) 4

13.9 The greatest value of $f(x) = \int_{5\pi/3}^{x} (6\cos t - 2\sin t)dt$ in the interval $\left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$, is equal to

(1) $3\sqrt{3} - 2\sqrt{2} - 1$ (2) $3\sqrt{3} + \sqrt{2} - 1$ (3) $3\sqrt{3} + 2\sqrt{2} + 1$ (4) None of these

13.10 Let $f : R \to R$ be a continuous which satisfies $f(x) = \int_0^x f(t)dt$. Then f(x) is

(1) non periodic function (2) not an odd function (3) f(2011) = 2011 (4) f(2011) = 0

13.11 $\lim_{n\to\infty} \frac{\sum_{r=1}^{n} r^{\frac{1}{a}}\left(n^{a-\frac{1}{a}} + r^{a-\frac{1}{a}}\right)}{n^{a+1}} =$

(1) a (2) $\frac{1}{a}$ (3) $a + \frac{1}{a}$ (4) 1

13.12 $\int_0^{\pi} \cos^n x\, dx$ (n is even integer) is equal to

(1) $\frac{\pi(2n)!}{2^{2n}(n!)^2}$ (2) $\frac{n!\,\pi}{2^n((n/2)!)^2}$ (3) $\frac{\pi\, n!}{2^{n+1}\left(\left(\frac{n}{2}\right)!\right)^2}$ (4) none of these

13.13 The value of integral

$\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)dx$

must be equal to

(1) $\frac{\pi}{2} + 1$ (2) $\frac{\pi}{2} - 1$ (3) $\pi + 1$ (4) $2\pi - 1$

13.14 $\int_{-2}^{5} \text{sgn}(x - [x])\,dx =$

(1) 0 (2) 1 (3) 9 (4) 7

13.15 $\int_0^{18} \frac{[x^2]dx}{[x^2 - 36x + 324] + [x^2]} =$

(1) 0 (2) 1 (3) 9 (4) 7

13.16 Total number of integral values of a such that $\int_a^0 (9^{-2t} - 2(9)^{-t}dt \le 0$ is equal to

(1) 0 (2) 1 (3) 9 (4) 7

13.17 $\int_{-n}^{n} (-1)^{[x]} dx$ when $n \in N$ = in all of these [.] denotes the greatest integer function

(1) 0 (2) 1 (3) 9 (4) 7

13.18 Let $F(x) = \int_x^{x+3} 2t(5-t)dt$. If M is the maximum value of F(x) then the sum of the digits of M is

(1) 6 (2) – 6 (3) 5 (4) 0

13.19 If $F(x) = \frac{\sin^2 x}{(1-e^{-3x})\sin^2 2x}$, then $\int_{-\pi/4}^{\pi/4} f(x)dx =$

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{5}$ (4) 0

13.20 The value of the integral $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$ is equal to

(1) π (2) 2π (3) 4π (4) none of these

13.21 If $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}=0$, where C_0, C_1, C_2 are all real, the equation $C_2x^2 + C_1x + C_0 = 0$ has:

(1) atleast one root in (0, 1) (2) one root in (1, 2) & other in (3, 4)

(3) one root in (–1, 1) & the other in (–5, –2) (4) both roots imaginary

13.22 Let f(x) = minimum (|x| , 1 – |x|, 1/4) , $\forall x \in R$, then the value of $\int_{-1}^{1} f(x)\, dx$ is equal to

(1) $\frac{1}{32}$ (2) $\frac{3}{8}$ (3) $\frac{4}{32}$ (4) none of these

13.23 Let $I_1 = \int_0^1 \frac{e^x dx}{1+x}$ and $I_2 = \int_0^1 \frac{x^2 dx}{e^{x^3}(2-x^3)}$, then $\frac{I_1}{I_2}$ is

(1) 3/e (2) e/3 (3) 3e (4) 1/3e

Level : II (Tough)

13.24 $\int_0^x \frac{2^t}{2^{[t]}}dt$, where [.] denotes the greatest integer function and $x \in R^+$, is equal to

(1) $\frac{1}{\ell n2}([x]+2^{[x]}-1)$ (2) $\frac{1}{\ell n2}\left(2^x-2^{[x]}+[x]\right)$ (3) $\frac{1}{\ell n2}([x]-2^{[x]})$ (4) $\frac{1}{\ell n2}([x]+2^{[x]}+1)$

13.25 A function f is continuous for all x . (and not every where zero)

such that $f^2(x) = \int_0^x f(t)\cdot\frac{\cos t}{2+\sin t}dt$, then f(x) is equal to

(1) $\frac{1}{2}\ell n\left(\frac{x+\cos x}{2}\right), x \neq 0$ (2) $\frac{1}{2}\ell n\left(\frac{3}{2+\cos x}\right), x \neq 0$

(3) $\frac{1}{2}\ell n\left(\frac{2+\sin x}{2}\right), x \neq n\pi, n \in I$ (4) $\frac{\cos x+\sin x}{2+\sin x}, x \neq \pi+\frac{3\pi}{4}, n \in I$

13.26 $\int_0^1 \frac{x^{\cos\alpha}-1}{\ell nx}dx$ where $\alpha \neq (2n+1)\pi$ is

(1) $\ell n(1 - \sin\alpha)$ (2) $\ell n(1 + \sin\alpha)$ (3) $\ell n(1 - \cos2)$ (4) $\ell n(1 + \cos\alpha)$

13.27 Let f (x) be differentiable function such that $f(x) = x^2 + \int_0^x e^{-t}f(x-t)dt$. Then

(1) f (x) is decreasing for $x \in (-\infty, -2] \cup [0, \infty)$ (2) f (x) is increasing for $x \in [-2, 0]$

(3) $f(1) = \frac{1}{3}$ (4) $f(1) = \frac{4}{3}$

13.28 $\int_{-\pi/2}^{\pi/2} [\tan x]dx$, (where [.] denotes the greatest integer function) is equal to .

(1) irrational number (2) positive real number (3) rational number (4) negative real number

13.29 $\int_0^{\pi/4} (\pi x - 4x^2)\ell n(1+\tan x)dx =$

(1) $\frac{\pi^3}{192}\ell n2$ (2) $\frac{\pi^3}{191}\ell n2$ (3) $\frac{\pi^3}{192}\ell n3$ (4) $\frac{\pi^2}{192}\ell n2$

13.30 Let f(x) = x + sinx and g(x) be the inverse function of f(x). Then $\int_0^{\pi} g(x)dx =$

(1) $\frac{\pi^2}{2}+2$ (2) $\frac{\pi^2}{2}-3$ (3) $\frac{\pi^2}{2}-2$ (4) $\frac{\pi^2}{2}$

13.31 If $\int_0^1 \tan^{-1} x dx = P$ and $\int_0^{\pi/4} \tan^{-1}\left(\frac{1}{\tan^2\theta - \tan\theta + 1}\right)\sec^2\theta d\theta$ = KP, then find the value of K.

(1) 5 (2) 3 (3) 7 (4) 2

SECTION - II : ASSERTION & REASONING TYPE

13.32 **Statement –1 :** $\int_0^1 \ell n(1+x)dx < \frac{1}{2}$

Statement –2 : ln (1 + x) < x in (0, 1)

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

13.33 Let $f(x) = \int_{-5}^{x} (t^2 - t + 2)(t^2 - t - 2)(t^2 - t - 6)(t^2 - t - 12)\,dt$

Statement 1 : The sum of values of x where f (x) is maximum is – 1

Statement 2 : If f ′(3) = 0 then f (x) has either maximum or minimum at x = c and x = c is not an inflection point.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

13.34 **Statement 1 :** $\int_{-\pi/2}^{\pi/2}\left(\frac{e^{|\sin x|}\cos x}{1+e^{\tan x}}\right)dx = 1-e$

Statement f 2 : If f (x) is an even function then $\int_{-\pi/2}^{\pi/2} f(x)\,dx = 2\int_0^{\pi/2} f(x)dx$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

13.35 **Statement -1 :** Let $I_n = \int_0^1 (1-x^5)^n dx$. Then $\frac{I_{10}}{I_{11}} = \frac{55}{54}$

Statement-2 : If u(x) and v(x) are differentiable functions then $\int uv\,dv = u\int v\,dx - \int\left(\frac{du}{dx}\int vdx\right)dx$.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

13.36 **Statement 1 :** $\int_0^1 \frac{\ell n x}{1+x}dx = -\int_0^1 \frac{\ell n(1+x)}{x}$

Statement 2 : If f(t) is an odd function then $\phi(x) = \int_a^x f(t)dt$ is an even function

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

13.37 **Statement 1 :** $\int_{-2}^{0}\{x^5+5x^4+10x^3+10x^2+5x+5+\tan^7(x+1)\}dx = 0$

Statement 2 : $\int_{-a}^{a}\frac{f(\sin x)}{f(\cos x)+f(\sin^2 x)}dx$ where f is an odd function = 0

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC **14**

AREA UNDER CURVE

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

14.1 The area bounded by the curve $2x^2 + y^2 = 2$ is -

(1) π (2) $\sqrt{2}\pi$ (3) $\frac{\pi}{2}$ (4) 2π

14.2 The area bounded by the curve $y^2(1 - x) = x^2(1 + x)$ between $x = 0$, $x = 1$ is -

(1) $\frac{\pi}{2} + 2$ (2) $2 - \frac{\pi}{2}$ (3) $\pi + 2$ (4) $\frac{\pi}{2} + 1$

14.3 The area bounded by the curve $y = xe^{-x^2}$, $y = 0$ and the maximum ordinate is

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{e}}$ (3) $\frac{1}{2}\left(1-\frac{1}{\sqrt{e}}\right)$ (4) $\frac{1}{2}\left(1+\frac{1}{e}\right)$

14.4 The area enclosed between the curve $y = \sin^2 x$ and $y = \cos^2 x$, $0 \le x \le \pi$ is -

(1) 1 sq. unit (2) $\frac{1}{2}$ sq. units (3) 2 sq. units (4) None of these

14.5 The area bounded by $y = \log_e x$, x-axis and the ordinate $x = e$ is given by

(1) 4 sq. units (2) $\frac{1}{2}$ sq. units (3) 1 sq. units (4) none of these

14.6 The area bounded by the curve $y = \sin^{-1}x$ and the lines $x = 0$, $|y| = \frac{\pi}{2}$

(1) 2 sq. units (2) 4 sq. units (3) 8 sq. units (4) 16 sq. units

14.7 The area of the figure bounded by the straight line $x = 0$, $x = 2$ and the curves $y = 2^x$, $y = 2x - x^2$ is

(1) $\left(\frac{4}{\ell n 2} - \frac{8}{3}\right)$ sq. units (2) $\left(\frac{4}{\ell n 2} + \frac{8}{3}\right)$ sq. units

(3) $\left(\frac{8}{\ell n 3} - \frac{4}{3}\right)$ sq. units (4) $\left(\frac{3}{\ell n 2} - \frac{4}{3}\right)$ sq. units

14.8 The area bounded by the curve $y = |x - 1|$ and $y = 3 - |x|$

(1) 4 sq. units (2) 2 sq. units (3) 6 sq. units (4) 8 sq. units

14.9 The area of the region $R = \{(x, y) : |x| \le |y| \text{ and } x^2 + y^2 \le 1\}$ is

(1) $\frac{3\pi}{8}$ sq. units (2) $\frac{5\pi}{8}$ sq. units (3) $\frac{\pi}{2}$ sq. units (4) $\frac{\pi}{8}$ sq. units

14.10 If y = f(x) make positive intercepts of 2 and 1 unit on x and y coordinate axes and encloses an area of $\frac{3}{4}$ square units with the axes, then $\int_0^2 x f'(x)\,dx$ is

(1) $\frac{3}{2}$ (2) 1 (3) $\frac{5}{4}$ (4) $-\frac{3}{4}$

14.11 The area of one curvilinear triangle formed by the curves y = sin x, y = cos x and x-axis

(1) $\left(2+\sqrt{2}\right)$ sq. units (2) $2-\sqrt{2}$ sq. units (3) $\left(\sqrt{2}-2\right)$ sq. units (4) none of these

14.12 The area bounded by the curve y = x|x|, x-axis and theordinates x = 1, x = – 1

(1) $\frac{4}{3}$ sq. units (2) $\frac{2}{3}$ sq. units (3) $\frac{1}{6}$ sq. units (4) $\frac{4}{5}$ sq. units

14.13 The area of the region lying between the line x – y + 2 = 0 and the curve $x = \sqrt{y}$

(1) $\frac{4}{3}$ sq. units (2) $\frac{2}{3}$ sq. units (3) $\frac{5}{3}$ sq. units (4) $\frac{10}{3}$ sq. units

14.14 The area enclosed between the curve $y^2 = x$ and y = |x|

(1) $\frac{1}{6}$ sq. units (2) $\frac{2}{3}$ sq. units (3) $\frac{5}{3}$ sq. units (4) $\frac{10}{3}$ sq. units

14.15 The area bounded by the parabola $x = y^2$ and the straight line y = 4 and y-axis

(1) $\frac{1}{6}$ sq. units (2) $\frac{64}{3}$ sq. units (3) $\frac{5}{3}$ sq. units (4) $\frac{32}{3}$ sq. units

14.16 The area inside the parabola $5x^2 - y = 0$ but outside the parabola $2x^2 - y + 9 = 0$ is

(1) $12\sqrt{3}$ (2) $6\sqrt{3}$ (3) $8\sqrt{3}$ (4) $8\sqrt{3}$

14.17 The area of the region on plane bounded by max (|x|, |y|) ≤ 1 and xy ≤ $\frac{1}{2}$ is

(1) 1/2 + ℓn 2 (2) 3 + ℓn 2 (3) 31/4 (4) 1 + 2 ℓn 2

14.18 The area bounded by $y = x^2$, y = [x + 1], x ≤ 1 and the y-axis is

(1) 1/3 (2) 2/3 (3) 1 (4) 7/3

14.19 The area contained between the curve $xy = a^2$, the vertical line x = a, x = 4a (a > 0) and x-axis is

(1) $a^2 \ell n\, 2$ (2) $2a^2 \ell n\, 2$ (3) $a\, \ell n\, 2$ (4) $2a\, \ell n\, 2$

14.20 The area of the closed figure bounded by the curves $y = \sqrt{x}$, $y = \sqrt{4-3x}$ and y = 0 is:

(1) $\frac{4}{9}$ (2) $\frac{8}{9}$ (3) $\frac{16}{9}$ (4) none

14.21 The area bounded by the curve $x^2 = 4y$, x-axis and the line x = 2 is

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 2

14.22 The area bounded by the parabola $y = 4x^2$, x = 0 and y = 1, y = 4 is

(1) 7 (2) $\frac{7}{2}$ (3) $\frac{7}{3}$ (4) $\frac{7}{4}$

14.23 The area bounded by the curve $y = \frac{1}{x^2}$ and its asymptote from x = 1 to x = 3 is

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{6}$

14.24 The area bounded by the curve $y^2 = 4x$ and the line $2x - 3y + 4 = 0$ is

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) $\frac{5}{3}$

Level : II (Tough)

14.25 If the curve $y = ax^{1/2} + bx$ passes through the point (1, 2) and lies above x-axis for $0 \le x \le 9$ and the area enclosed by the curve, the x-axis and the line x = 4 is 8 sq. units, then

(1) a = 1, b = 1 (2) a = 3 b = – 1 (3) a = 3, b = 1 (4) a = 1, b = –1

14.26 The area enclosed by the curve $x = a \sin^3 t$ and $y = a \cos^3 t$ is given by

(1) $12a^2 \int_0^{\pi/2} \cos^4 t \sin^2 t \, dt$ (2) $12a^2 \int_0^{\pi/2} \cos^2 t \sin^4 t \, dt$

(3) $2 \int_{-a}^{a} (a^{2/3} - x^{2/3})^{3/2} dx$ (4) all of these

14.27 The parabola $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinates axes. If S_1, S_2, S_3 are the areas of these parts numbered from top to bottom, respectively, then–

(1) $S_1 : S_2 : S_3 = 1 : 1 : 1$ (2) $S_1 : S_2 : S_3 = 1 : 2 : 3$

(3) $S_1 : S_2 : S_3 = 3 : 2 : 1$ (4) $S_1 : S_2 : S_3 = 1 : 2 : 4$

14.28 If A_i is the area bounded by $|x - a_i| + |y| = b_i$ $i \in N$, where $a_{i+1} = a_i + \frac{3}{2} b_i$ and $b_{i+1} = \frac{b_i}{2}$, $a_1 = 0$ and $b_1 = 32$, then

(1) $A_3 = 64$ (2) $A_3 = 256$

(3) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{8}{3}(32)^2$ (4) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{4}{3}(16)^2$

14.29 Find the area enclosed between the curve $y^2 (2a - x) = x^3$ and the line x = 2a above x-axis

(1) $\frac{5\pi a^2}{2}$ (2) $\frac{3\pi a^2}{2}$ (3) $\frac{\pi a^2}{2}$ (4) $\frac{3\pi a}{2}$

14.30 The area enclosed by the curve $y = \sqrt{4 - x^2}$, $y \ge \sqrt{2} \sin \frac{\pi x}{2\sqrt{2}}$ and x-axis is divided by the y-axis in the ratio

(1) $\frac{2\pi^2}{2\pi + \pi^2 - 12}$ (2) $\frac{2\pi^2}{2\pi + \pi^2 - 8}$ (3) $\frac{\pi}{2\pi + \pi^2 - 8}$ (4) $\frac{2\pi^2}{\pi + \pi^2 - 8}$

14.31 The area cut off a parabola by any double ordinate is k times the corresponding rectangle contained by that double ordinate and its distance from the vertex, then find the value of k.

(1) 2/5 (2) 4/3 (3) 2/3 (4) 5/2

14.32 The slope of the tangent to the curve y = f(x) at (x, f(x)) is 2x + 1. If the curve passes through the point (1, 2). Then find the area of the region bounded by the curve, the x-axis and the line x = 1.

(1) $\frac{5}{7}$ (2) $\frac{5}{6}$ (3) $\frac{1}{6}$ (4) $\frac{2}{5}$

SECTION - II : ASSERTION & REASONING TYPE

14.33 **Statement-1 :** The area bounded by parabola $y = x^2 - 4x + 3$ and $y = 0$ is $\frac{4}{3}$ sq. units

Statement-2 : The area bounded by the curve $y = f(x) \geq 0$ and $y = 0$ between the ordinates $x = a$ and $x = b$, where $b \geq a$) is $\int_a^b f(x)\,dx$

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True,Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

14.34 **Statement-1 :** The area of the plane region bounded by the curve $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is $\frac{4}{3}$

Statement-2 : The area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$ is $\frac{9}{2}$

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

14.35 **Statement-1 :** Area bounded by parabola $y = x^2 - 4x + 3$ and $y = 0$ is 4/3 sq. units.
Statement-2 : Area bounded by curve $y = f(x) \geq 0$ and $y = 0$ between ordinates $x = a$ and $x = b$ ($b > a$) is $\int_a^b f(x)\,dx$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

14.36 **Statement-1 :** Area bounded by $y = \tan x$, $y = \tan^2 x$ in between $x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ is equal to $\left(\frac{\pi}{4} + \ell n\sqrt{2} - 1\right)$.

Statement-2 : Area bounded by $y = f(x)$ and $y = g(x)$ $\{f(x) > g(x)\}$ between $x = a$, $x = b$ is $\int_a^b (f(x) - g(x))\,dx$. ($b > a$)

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

14.37 **Statement-1 :** Area formed by curve $y = \cos x$ with $y = 0$, $x = 0$ and $x = \frac{3\pi}{4}$ is $2 - \frac{1}{\sqrt{2}}$

Statement-2 : Area of curve $y = f(x)$ with x-axis between ordinates $x = a$ and $x = b$ is $\int_a^b f(x)\,dx$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 15 DIFFERENTIAL EQUATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

15.1 Solution of the differential equation $(x\cos x - \sin x + yx^2)dx + x^3dy = 0$ is -

(1) $\frac{\sin x}{x} + xy = c$ (2) $\frac{\sin x}{x} + x = c$ (3) $\frac{\sin x}{x} + y = c$ (4) None of these

15.2 The co-ordinates of the focus of the conic satisfying the differential equation.

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$, $y(0) = 2$, $y'(0) = 4$ is -

(1) $\left[\frac{15}{4}, 0\right]$ (2) $\left[-\frac{1}{4}, 0\right]$ (3) $\left[\frac{7}{4}, 0\right]$ (4) $\left[\frac{-15}{4}, 0\right]$

15.3 The differential equation satisfied by all the circle in the x - y plane is $(1 + y_1^2)y_3 = \lambda y_1 y_2^2$ where λ =

(1) 1 (2) 2 (3) –2 (4) 3

15.4 The order of the differential equation whose general solution is given by $y = a\cos x + b\sin x + ce^{-x}$ is

(1) 1 (2) 2 (3) 3 (4) 4

15.5 The order and degree of the differential equation $\left[4 + \left(\frac{dy}{dx}\right)^2\right]^{2/3} = \frac{d^2y}{dx^2}$ are equal to

(1) 2, 2 (2) 3,3 (3) 2, 3 (4) 3, 2

15.6 The general solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

(1) $e^y = e^x + \frac{x^3}{3} + c$ (2) $e^y = e^x + 2x + c$ (3) $e^y = e^x + x^3 + c$ (4) none of these

15.7 The general solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$ is

(1) $\tan^{-1}\left(\frac{x}{y}\right) = \ell ny + c$ (2) $\tan^{-1}\left(\frac{y}{x}\right) = \ell nx + c$ (3) $\tan^{-1}\left(\frac{x}{y}\right) = \ell nx + c$ (4) $\tan^{-1}\left(\frac{y}{x}\right) = \ell ny + c$

15.8 The solution of the differential equation $\frac{dy}{dx} = \frac{3y - 7x - 3}{3x - 7y + 7}$ is

(1) $(y - x - 2)^5 (y + x - 5)^7 = c$ (2) $(y - x - 5)^2 (y + x - 1)^7 = c$
(3) $(y - x - 7)^2 (y + x - 5) = c$ (4) $(y - x - 1)^2 (y + x - 1)^5 = c$

15.9 The solution of the differential equation $(1 + y^2)dx = (\tan^{-1} y - x)\, dy$ is

(1) $xe^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$ (2) $xe^{\tan^{-1}y}(\tan^{-1}y + 1) = c$

(3) $xe^{\tan^{-1}y}(\tan^{-1}y - 1) = c$ (4) None of these

15.10 The solution of the differential equation $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is

(1) $xy^2 = y^5 + c$ (2) $xy^2 + 2y^5 = c$ (3) $xy^2 = 2y^5 + c$ (4) none of these

15.11 If m and n are the order & degree of the equation $\left(\frac{d^2y}{dx^2}\right)^5 + 4.\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$, then

(1) m = 3, n = 2 (2) n = 2, m = 5 (3) m = 2, n = 3 (4) n = 5, m = 2

15.12 The differential equation of the family of all parabolas whose axis is x-axis are of

(1) first degree (2) second degree (3) third degree (4) fourth degree

15.13 The solution of the equation $\frac{dy}{dx} = e^{y+x} + e^{y-x}$ is

(1) $e^y(e^x + e^{-x} + c) = -1$ (2) $e^y(e^x - e^{-x} + c) = -1$

(3) $e^x(e^x - e^{-x} + c) = -1$ (4) $e^{x+y}(e^{2x} + ce^x - 1) = -1$

15.14 The solution of the differential equation $(x + y)dy - (x - y)\, dx = 0$ is

(1) $y^2 + 2xy + x^2 = k$ (2) $y^2 + 2xy - x^2 = k$ (3) $y^2 - 2xy + x^2 = k$ (4) none of these

15.15 The solution of the differential equation $\frac{xdy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is

(1) $x\sin^{-1}\left(\frac{y}{x}\right) + c = 0$ (2) $x\sin y + c = 0$ (3) $x\sin\left(\frac{y}{x}\right) = 0$ (4) $y = x\sin^{-1}\left(\frac{c}{x}\right)$

15.16 The orthogonal trajectory of $x^{2/3} + y^{2/3} = a^{2/3}$ is

(1) $y^{4/3} + x^{4/3} = c$ (2) $y^{4/3} - x^{4/3} = c$ (3) $x^{2/3} - y^{2/3} = c$ (4) $(xy)^{4/3} = c$

15.17 A curve passes through (2, 0) and the slope of the tangent at P(x, y) is equal to $\frac{(x+1)^2 + y - 3}{x+1}$ then the equation of the curve is

(1) $y = x^2 - 2x$ (2) $y = x^3 - 8$ (3) $y^2 = x^2 + 2x$ (4) none of these

15.18 The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x - 2y + 5}{2x - y + 4}\right)$ is

(1) $(x + y - 1)^3 = A(x - y + 3)$ (2) $(x + y + 3) = B(x + y - 1)^3$

(3) $(x + y - 3) = C(x - y + 1)^3$ (4) $(x + y - 1)^3 = D(x + y - 3)$

15.19 If $\frac{xdy}{dx} + y = x\frac{f(xy)}{f'(xy)}$ then f(xy) is equal to

(1) $ke^{x^2/2}$ (2) $ke^{y^2/2}$ (3) $ke^{(xy)^2/2}$ (4) $ke^{xy/2}$

15.20 The differential equation $y'' + \frac{2}{1-y}(y')^2 = 0$

(1) is linear

(2) has a solution $y = e^x + c_1$

(3) has a solution $y = c_1 - c_2 x^2$

(4) has a solution $y = \frac{x+c_1}{x+c_2}$

Level : II (Tough)

15.21 Solution of the differential equation $x^2y\frac{d^2y}{dx^2} + \left(x\frac{dy}{dx} - y\right)^2 = 0$ is

(1) $y = \sqrt{x(c_2x^2 + 2c_1)}$ (2) $y = \sqrt{x(c_1 - 2c_2x^2)}$ (3) $y = \sqrt{x(c_2x - 2c_1)}$ (4) $y^2 = \sqrt{x(c_2x + 2c_1)}$

15.22 The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$ is

(1) $xe^{2\tan^{-1}y} = e^{\tan^{-1}y} + c$

(2) $(x - 2) = ce^{\tan^{-1}y}$

(3) $2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c$

(4) $xe^{\tan^{-1}y} = \tan^{-1}y + c$

15.23 The solution of the differential equation $\frac{x}{x^2+y^2}\,dy = \left(\frac{y}{x^2+y^2} - 1\right)dx$ is

(1) $y = x\cot(c-x)$ (2) $\cos^{-1}\left(\frac{y}{x}\right) = (-x + c)$ (3) $y = x\tan(c-x)$ (4) $\left(\frac{y^2}{x^2}\right) = x\tan(c-x)$

15.24 Solution of the differentiable equation

$x = 1 + xy\left(\frac{dy}{dx}\right) + \frac{(xy)^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{(xy)^3}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots\ldots$ is

(1) $y = \log_e x + c$

(2) $y = (\log_e x)^2$

(3) $y = \pm\sqrt{(\log_e x)^2 + 2c}$

(4) $xy = x^y + k$

15.25 Solution of the differentiable equation $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2}\right)$ when $y = 1$, is

(1) $\log\left|\frac{(x-y)^2-2}{2}\right| = 2(x+y)$

(2) $\log\left|\frac{(x-y)^2+2}{2}\right| = 2(x-y)$

(3) $\log\left|\frac{(x+y)^2+2}{2}\right| = 2(x-y)$

(4) None of these

SECTION - II : ASSERTION & REASONING TYPE

15.26 **Statement-1 :** The differential equation $y^3\,dy + (x + y^2)\,dx = 0$ becomes homogeneous if we put $y^2 = t$

Statement-2 : All differential equation of first order and first degree becomes homogeneous if we put $y = tx$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

15.27 **Statement-1 :** The integrating factor of the differential equation in $\frac{dy}{dx}(x\ell nx) + y = 2\ell nx$ is ℓnx

Statement-2 : The general solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ is $y = A \sin (x + B)$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

15.28 **Statement – 1 :** The curve $y = cx^2$, c being any arbitrary constant intersects the curves $x^2 + 2y^2 = 2c$ at right angle.

Statement – 2 : As above curves traces orthogonal trajectory.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

15.29 **Statement – 1 :** A ray of light from origin after reflection at the point P(x,y) of any curve becomes parellel to x–axis, the equation of curve may be $y^2 = 2x+1$

Statement – 2 : A ray of light parallel to axis after reflection from parabolic mirror always passes through the focus.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

15.30 **Statement – 1 :** The differential equation of the family of hyperbolas with asymptotes as the line $x + y = 1$ & $x - y = 1$ is $(x-1) = y\frac{dy}{dx}$

Statement – 2 : As , the eccentricity of the rectangular hyperbola is $\sqrt{2}$.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 16

QUADRATIC EQUATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

16.1 If α, β are roots of the equation $x^2 + px + q = 0$ then the equation whose roots are $(\alpha + p)^{-2}$ are $(\beta + p)^{-2}$ is -
(1) $q^2x^2 - (p^2 - 2q)x + 1 = 0$
(2) $x^2 - (p^2 - 2q)x + q^2 = 0$
(3) $q^2x^2 + (p^2 - 2q)x + 1 = 0$
(4) None of these

16.2 If one root of equation $x^2 + Ax + 12 = 0$ is 4 and the root of equation $x^2 + 2Ax + B = 0$ are equal then value of B is -
(1) 49 (2) 4 (3) $\frac{4}{29}$ (4) $\frac{49}{4}$

16.3 If roots of the euation $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}\,x + q = 0$ are simultaneously real, then
(1) $p = q,\ r \neq 0$ (2) $2q = \pm\sqrt{pr}$ (3) $\frac{p}{q} = \frac{q}{r}$ (4) $q = r \neq 0$

16.4 If both roots of quadratic equation $2x^2 + \lambda x - (\lambda + 1) = 0$ are greater then 1 then λ lies completely in intervel
(1) $(-\infty, -10)$ (2) $(-4, \infty)$ (3) $(-6, -1)$ (4) $(-\infty, 0)$

16.5 If equation $(p^2 + p - 1)x^2 - (2p^2 - 4)x - (3p^2 + 9p + 3) = 0$ is identity in p, then find the value of x -
(1) 3 (2) –1 (3) 1 (4) – 3

16.6 If equation $(r^2 - 2r + 1)x^2 + (r^2 - 3r + 2)x - (r^2 + 4r + 3) = 0$ is identity in x then find value of r -
(1) 1 (2) 2 (3) – 1 (4) not possible

16.7 If α, β are roots of quadratic equation $2x^2 - 5x - 7 = 0$, then equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is
(1) $x^2 + 53x + 14 = 0$
(2) $14x^2 + 53x + 14 = 0$
(3) $14x^2 + 25x + 14 = 0$
(4) none of these

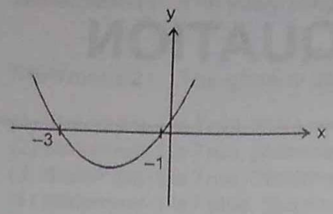
16.8 If ℓ, m, n are real and positive and $\ell \neq m$, then roots of $(\ell - m)x^2 - 5(\ell + m)x - 2(\ell - m) = 0$ are
(1) real and equal
(2) Imaginary
(3) real and unequal
(4) none of these

16.9 If roots of the equation $ax^2 + bx + c = 0$ are imaginary then roots of the equation $(4c + 2b + a)x^2 - 2(a + b)x + a = 0$ are -
(1) real and equal (2) imaginary (3) real and unequal (4) none of these

16.10 If a is purely imaginary constant then roots of the equation $x^2 + 2ax + 2a^2 = 0$ are
(1) real and rational (2) irrational (3) imaginary (4) none of these

16.11 If one root of equation $x^2 + 3x + 4 = 0$ and $2x^2 + ax + 8 = 0$ is common, then value of a is -
(1) 3 (2) 6 (3) 4 (4) 2

16.12 Graph of quadratic expression $y = ax^2 + bx + c$ is given, then



(1) $a + b + c < 0$ (2) $(4a + c)^2 - (2b)^2 < 0$
(3) $(4a + c)^2 - (2b)^2 > 0$ (4) $9a - 3b + c > 0$

16.13 Find interval of x which satisfying $x^2 - 2 < x^2 + x + 3 < (x - 2)^2 - 6$
(1) $(-5, \infty)$ (2) $(-5, -1)$ (3) $(-\infty, -1)$ (4) None of these

16.14 Value of λ so that roots of quadratic equation $4x^2 - (\lambda + 2)x + (\lambda^2 - 5\lambda + 6) = 0$ are of opposite sign –
(1) $(-2, 2)$ (2) $(-2, 3)$ (3) $(2, 3)$ (4) None of these

16.15 Value of λ so that quadratic equation $x^2 + (\lambda + 1)x + (\lambda^2 - 3\lambda - 6) = 0$ has one root lies between 1 and 2 and and another root lies between 2 and 3 –
(1) $(0, 1)$ (2) $(1-\sqrt{5}, 1+\sqrt{5})$ (3) $(1, 1+\sqrt{5})$ (4) ϕ

16.16 If $2x^2 + y^2 - 2xy - 4y + 8 = 0$ then pair (x, y) is -
(1) (2, 4) (2) (8, 4) (3) (4, 2) (4) none of these

16.17 If $x + y$ and $2x - y$ are factors of $\lambda x^3 - x^2y + \mu xy^2 + y^3$, $x, y \neq 0$, then pair (λ, μ) is -
(1) (–2, 2) (2) (2, 2) (3) (2, –2) (4) none of these

16.18 Value of x satisfying $||x - 2| - 3| < 4$ are -
(1) (1, 3) (2) (–2, 1) (3) (3, 6) (4) (–2, 6)

16.19 Value of k so that equation $(x^2 - 2x)^2 - 3(x^2 - 2x) + (k + 2) = 0$ has two real solution -
(1) $(-\infty, -6)$ (2) $\left\{\frac{1}{4}\right\}$ (3) $(-\infty, -6) \cup \left\{\frac{1}{4}\right\}$ (4) none of these

Level : II (Tough)

16.20 If α, β are roots of $x^2 + px + 7 = 0$ and γ, δ are roots of $x^2 + px - 4 = 0$ then value of $(\alpha - \gamma)(\alpha - \delta)$ is
(1) 3 (2) 11 (3) – 11 (4) – 3

16.21 α, β are roots of $x^2 - 6x + 4 = 0$ then value of $(\alpha - 6)^{-2} + (\beta - 6)^{-2}$ is –
(1) $\frac{7}{4}$ (2) 2 (3) 4 (4) $\frac{7}{2}$

16.22 If $a \neq 0$ and one root of the equation $2x^2 + 3x + a = 0$ is double of one root of equation $2x^2 + 9x + 4a = 0$, then a is
(1) 2 (2) 3 (3) 1 (4) – 1

16.23 Range of quadratic expression $y = x^2 + 6x - 4$, $x \in (-4, 3)$ is
(1) $(-12, 23)$ (2) $(-13, 23)$ (3) $[-13, 23]$ (4) $[-13, 23)$

16.24 Value of x which satisfy $\log_{1/4}[\log_2(x + 2)] > 0$ and $|x - 1| + |x - 2| < 2$ is -
(1) $(-1, 0)$ (2) $(-1, 0) \cup \left(\frac{1}{2}, \frac{5}{2}\right)$ (3) $\left(\frac{1}{2}, \frac{5}{2}\right)$ (4) ϕ

16.25 Roots of the cubic equation $x^3 - 9x^2 + px - 27 = 0$ are positive & real then value of p is -
(1) 27 (2) 9 (3) 18 (4) 36

16.26 If α, β, γ are roots of $x^3 - 5x^2 + x - 2 = 0$, then value of $\left(\frac{\alpha+2}{\alpha-2}\right)\left(\frac{\beta+2}{\beta-2}\right)\left(\frac{\gamma+2}{\gamma-2}\right)$ is -
(1) 2 (2) $\frac{1}{2}$ (3) $\frac{3}{8}$ (4) $\frac{8}{3}$

16.27 Value of k so that equation $(x^2 - 2x)^2 - 3(x^2 - 2x) + (k + 2) = 0$ has four real solution
(1) $\left(-2, \frac{1}{4}\right)$ (2) $\left(-6, \frac{1}{4}\right)$ (3) (–6, – 2) (4) not possible

16.28 If equation $x^2 + px + q = 0$ and equation $x^2 - rx + s = 0$ have a common root $\alpha = 1$ then
(1) $p + q + r = 2s$ (2) $p + q + r = 3s$ (3) $p + q + r + s = 0$ (4) $p + q + r = s$

SECTION - II : ASSERTION & REASONING TYPE

16.29 **Statement-1 :** Equation $(x - p)(x - r) + \lambda (x - q)(x - s) = 0$, $p < q < r < s$ has non zero real roots if $\lambda > 0$
Statement-2 : Equation $ax^2 + bx + c = 0$, $a,b,c \in R$ has non zero real roots if $b^2 - 4ac < 0$
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

16.30 **Statement-1 :** If roots of equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$
Statement-2 : If a,b,c are odd integer then roots of the equation $4abc\, x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

16.31 **Statement-1 :** Number of values of 'a' for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x, is 2.
Statement-2 : If $a = b = c = 0$, then equation $ax^2 + bx + c = 0$ is an identity in x.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

16.32 **Statement-1 :** If $b^2 - 4ac < 0$ then roots of equation $ax^2 + bx + c = 0$ are non-real.
Statement-2: Equation $ix^2 - 3ix + 2i = 0$ has non-real root as $b^2 - 4ac = 9i^2 - 4i(2i) = -1$ is negative.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 17 SEQUENCE & SERIES

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

17.1 If equation $px^2 + qx + r = 0$ where 2p, q, 2r are in G.P. roots of $(\alpha^2, 4\alpha - 4)$, then find the value of $2p + 4q + 7r$ is

(1) 0 (2) 10 (3) 14 (4) 18

17.2 If sum of first m terms of an A.P. is zero, then sum of next n terms (a being the first terms) is –

(1) $\frac{-am(m+n)}{m+1}$ (2) $\frac{-an(m+n)}{m+1}$ (3) $\frac{-an(m+n)}{n+1}$ (4) $\frac{-am(m+n)}{n-1}$

17.3 If a, b, c are in G.P. and equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$

have a common root, then $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

(1) AP (2) GP (3) HP (4) None of these

17.4 A student read common difference of an A.P. as –2 instead of 2 and got the sum of first 5 terms as –5. Actual sum of first five terms is -

(1) 25 (2) –25 (3) –35 (4) 35

17.5 If there are six letters $L_1, L_2, L_3, L_4, L_5, L_6$ and their corresponding six envelopes $E_1, E_2, E_3, E_4, E_5, E_6$. Letters having odd value can be put into odd value envelopes and even value letters can be put into even value envelopes, so that no letter go into the right envelopes, the number of such arrangement will be equal to

(1) 1 (2) 2 (3) 3 (4) 4

17.6 The largest term common to the sequences

1, 11, 21, 31,....... to 100 terms and 31, 36, 41, 46,...... to 100 terms

(1) 381 (2) 471 (3) 281 (4) none of these

17.7 If S_r denote the sum of first r terms of an A.P. then $\frac{S_{3r} - S_{r-1}}{S_{2r} - S_{2r-1}}$ is equal to

(1) 2r – 1 (2) 2r + 1 (3) 4r + 1 (4) 2r + 3

17.8 Let $t_r = 2^{\frac{r}{2}} + 2^{\frac{-r}{2}}$. Then $\sum_{r=1}^{10} t_r^2$ is equal to

(1) $\frac{2^{21}-1}{2^{10}} + 20$ (2) $\frac{2^{21}-1}{2^{10}} + 19$ (3) $\frac{2^{21}-1}{2^{20}} - 1$ (4) None of these

17.9 If $(1 + x)(1 + x^2)(1 + x^4)........(1 + x^{128}) = \sum_{r=0}^{n} x^r$ then n is

(1) 255 (2) 127 (3) 60 (4) 81

17.10 If the 20^{th} term of an H.P. is 1 and the 30^{th} term is $\frac{-1}{17}$ then its largest term

(1) 1 (2) 3 (3) 0.5 (4) 7

17.11 The harmonic mean between two numbers is $\frac{21}{5}$, their A.M. 'A' and G.M. 'G' satisfy the relation $3A + G^2 = 36$. Then the sum of the squares of the numbers is

(1) 42 (2) 58 (3) 76 (4) 84

17.12 If eleven A.M's are inserted between 28 and 10 then the number of integral A.M.'s

(1) 4 (2) 5 (3) 6 (4) 7

17.13 The sum of 4 'G.M.' between 2 and 486

(1) 240 (2) 300 (3) 170 (4) 80

17.14 The product of 4 harmonic mean between $\frac{2}{3}$ and $\frac{2}{13}$

(1) $\frac{16}{5.7.9.11}$ (2) $\frac{32}{5.7.9.11}$ (3) $\frac{64}{5.7.9.11}$ (4) $\frac{8}{5.7.9.11}$

17.15 Find the greatest value of x^2y^3 where x and y are in the first quadrant and on the line $3x + 4y = 5$

(1) $\frac{5}{16}$ (2) $\frac{9}{16}$ (3) $\frac{3}{16}$ (4) $\frac{15}{32}$

17.16 The minimum value of $4^{\sin^2 x} + 4^{\cos^2 x}$

(1) 2 (2) $2\sqrt{2}$ (3) 4 (4) 1

17.17 If a, b, c, d $\in R^+$ and a, b, c, d are in H.P. then

(1) $a + d \geq b + c$ (2) $a + b \geq c + d$ (3) $a + c \geq b + d$ (4) none of these

17.18 If a, b, c $\in R^+$ then $(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is always

(1) ≥ 12 (2) ≥ 9 (3) ≤ 12 (4) None of these

17.19 The sum $2 + 5 + 10 + 17 + 26 + \ldots\ldots$ is equal to

(1) $\frac{n}{6}(2n^2 + 3n + 7)$ (2) $\frac{n}{6}(n^2 + 5n + 8)$ (3) $\frac{n}{12}(3n^2 + 4n + 9)$ (4) $\frac{n}{12}(3n^2 + 8n + 3)$

17.20 The sum $1 + 4 + 13 + 40 + 121 + \ldots\ldots\ldots$ is equal to

(1) $\frac{1}{4}[3^n - 2n - 3]$ (2) $\frac{1}{4}[3^{n+1} - 2n - 3]$ (3) $\frac{1}{4}[3^n - 2n - 2]$ (4) $\frac{1}{4}[3^{n+1} - 4n - 3]$

17.21 The sum of the series $1.n + 2(n - 1) + 3(n - 2) + \ldots\ldots + (n - 1)2 + n.1$

(1) $\frac{n(n+1)^2}{6}$ (2) $\frac{n(n+1)(n+4)}{6}$ (3) $\frac{n(n+1)(n+2)}{6}$ (4) $\frac{n(n+1)^2}{12}$

17.22 The sum to n-terms of the series $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots..$

(1) $\frac{n}{2n+1}$ (2) $\frac{n}{2(n+1)}$ (3) $\frac{n}{2(2n+1)}$ (4) $\frac{n+1}{(n+2)}$

17.23 The sum to infinite terms of the series when $|x| < 1$
$1 - 3x + 5x^2 - 7x^3 + ...$

(1) $\frac{1-x}{(1+x)^2}$ (2) $\frac{1-2x}{(1+x)^2}$ (3) $\frac{1}{(1+x)^2}$ (4) $\frac{1}{1+x}$

17.24 The sum of n-terms of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + ...$

(1) $\frac{5}{4} + \frac{15}{16}\left(1-\frac{1}{5^{n-1}}\right) - \frac{(3n-2)}{4(5^{n-1})}$ (2) $\frac{5}{4} + \frac{15}{16}\left(1-\frac{1}{5^{n}}\right) - \frac{(3n+1)}{4.5^{n}}$

(3) $\frac{5}{4} + \frac{15}{16}\left(1-\frac{1}{5^{n-2}}\right) - \frac{(3n-5)}{4.5^{n}}$ (4) $\frac{5}{4} + \frac{5}{16}\left(1-\frac{1}{5^{n-2}}\right) - \frac{(3n-2)}{5^{n-2}}$

17.25 The sum $\sum_{r=1}^{n} \frac{r}{(r+1)!}$

(1) $\frac{1}{(n+1)!}$ (2) $1 - \frac{1}{(n+1)!}$ (3) $1-\frac{1}{n!}$ (4) None of these

Level : II (Tough)

17.26 If a, b, c are in G.P. and a – b, c – a and b – c are in H.P. then the value of a + 4b + c

(1) 1 (2) 10 (3) 0 (4) 3

17.27 If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$, then the value of $S_n = 1 + \frac{3}{2} + \frac{5}{3} + ... + \frac{99}{50}$

(1) $H_{50} + 50$ (2) $100 - H_{50}$ (3) $49 + H_{50}$ (4) $H_{50} + 100$

17.28 If a, b, c $\in R^+$ then $\frac{bc}{b+c} + \frac{ac}{a+c} + \frac{ab}{a+b}$

(1) $\leq \frac{1}{2}(a + b + c)$ (2) $\geq \frac{1}{3}\sqrt{abc}$ (3) $\leq \frac{1}{3}(a + b + c)$ (4) $\geq \frac{1}{2}\sqrt{abc}$

17.29 If $\sum_{r=1}^{n} I(r) = n(2n^2 + 9n + 13)$ then the value of $\sum_{r=1}^{n} \sqrt{I(r)}$

(1) $\sqrt{\frac{3}{2}}(n^2 + 3n)$ (2) $\sqrt{\frac{5}{2}}(n^2 + n)$ (3) $\sqrt{\frac{5}{2}}(n^2 + 3n)$ (4) $\frac{1}{2}(n^2 + 3n + 2)$

17.30 If a, b, c are in H.P., then the value of $\frac{(ac+ab-bc)(ab+bc-ac)}{(abc)^2}$ is

(1) $\frac{(a+c)(3a-c)}{4a^2c^2}$ (2) $\frac{2}{bc} - \frac{1}{b^2}$ (3) $\frac{2}{ac} - \frac{1}{b^2}$ (4) $\frac{(a-c)(3a+c)}{4a^2c^2}$

17.31 Let $a_1, a_2, a_3, ..., a_n$ be a geometric progression with first term 'a' and common ratio r, then sum of $\frac{1}{a_1^2 - a_2^2} + \frac{1}{a_2^2 - a_3^2} + ... + \frac{1}{a_{n-1}^2 - a_n^2}$ is,

(1) $\frac{r^2(1-r^{2n-2})}{a^2r^{2n-2}(1-r^2)^2}$ (2) $\frac{r(1-r^{n-1})}{ar^{n-1}(1-r)^2}$ (3) $\frac{(1-r^{n-1})}{a^2r^{2n}(1-r)}$ (4) none of these

SECTION - II : ASSERTION & REASONING TYPE

17.32 **Statement-1 :** If the arithmetic mean of two numbers is $\frac{5}{2}$, geometric mean of the numbers is 2, then the harmonic mean will be $\frac{8}{5}$.

Statement-2 : For a group of positive numbers $(GM)^2 = (AM)(HM)$

(1) Statement-1 is true, statement-2 is true, statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true, statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

17.33 Let a, b, c be three distinct non-zero real numbers

Statement-1 : If a, b, c are in A.P. and b, c, a are in GP then c, a, b are in HP

Statement-2 : If a, b, c are in AP and b, c, a are in GP then a : b : c = 4 : 2 : –1

(1) Statement-1 is true, statement-2 is true, statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true, statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

17.34 **Statement-1 :** If x > 1, the sum $1 + 3\left(1-\frac{1}{x}\right) + 5\left(1-\frac{1}{x}\right)^2 + 7\left(1-\frac{1}{x}\right)^3 + \ldots\ldots\infty$ is $2x^2 - x$

Statement-2 : If $0 < y < 1$, the sum of the series $1 + 3y + 5y^2 + 7y^3 + \ldots\ldots\ldots$ is $\frac{1+y}{(1-y)^2}$

(1) Statement-1 is true, statement-2 is true, statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true, statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

sum of

TOPIC 18 BINOMIAL THEOREM

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

18.1 If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by -
(1) 113 (2) 123 (3) 133 (4) None of these

18.2 The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^{21}$ is
(1) T_{22} (2) T_{21} (3) T_{23} (4) T_{20}

18.3 If a, b, c represent the sides and A, B, C represent the angles of a ΔABC, then the value of the expression $\sum_{r=0}^{n} {}^nC_r\, a^r . b^{n-r} . \cos(rB - (n-r)A)$ is equal to:
(1) b^2 (2) c^n (3) a^n (4) None of these

18.4 In the expansion of $\left(2^x+\frac{1}{4^x}\right)^n$, $n \in N$ if sum of the coefficients of 2nd and 3rd term is 36, then which of these are correct ?
(1) n = 8
(2) n = 9
(3) $\frac{T_3}{T_2} = \frac{7}{4}$ when $x = -\frac{1}{3}$
(4) $\frac{T_3}{T_2} = 7$ when $x = \frac{1}{3}$

18.5 The coefficient of x^{160} in the expansion of $(x^8+1)^{60}\left(x^{12}+3x^4+\frac{3}{x^4}+\frac{1}{x^{12}}\right)^{-10}$ is
(1) ${}^{30}C_5$ (2) ${}^{30}C_6$ (3) ${}^{30}C_{24}$ (4) ${}^{30}C_{26}$

18.6 The number of dissimilar terms in the expansion of $\left[{}^4C_0 a^4b^2 + {}^4C_1 a^3b^{3/2} + {}^4C_2 a^2b + {}^4C_3 ab^{1/2} + {}^4C_4\right]^{20}$
(1) 61 (2) 81 (3) 41 (4) None of these

18.7 The number of rational terms in the expansion of $\left(\sqrt{2}+3^{1/3}\right)^{100}$ is
(1) 34 (2) 51 (3) 17 (4) 16

18.8 The greatest coefficient of x in the expansion of $(3+2x)^{50}$ is
(1) ${}^{50}C_{25} . 3^{25} . 2^{25}$ (2) ${}^{50}C_{25}$ (3) ${}^{50}C_{20} \times 3^{30} \times 2^{20}$ (4) none of these

18.9 The numerically greatest term in the expansion of $(3-4x)^{17}$, when $x = \frac{3}{2}$, is
(1) T_{12} (2) T_{13} (3) both T_{12} and T_{13} (4) T_9

18.10 The remainder when 3^{91} divided by 80 is
(1) 3 (2) 1 (3) 80 (4) 27

18.11 The remainder when $x = 5^{5^{5^5}}$ is divided by 24 is
(1) 1 (2) 3 (3) 5 (4) 23

18.12 If $(1 + x)^n = C_0 + C_1x + C_2x^2 + ...+ C_nx^n$, then $2C_0 + 2^2 . \frac{C_1}{2} + 2^3 . \frac{C_2}{3} + ... + 2^{n+1}\frac{C_n}{n+1} =$
(1) $\frac{3^{n+1}-1}{n+1}$ (2) $\frac{3^n-1}{n}$ (3) $\frac{3^{n+1}}{n+1}$ (4) $\frac{3^{n+2}-1}{n+2}$

18.13 $20 . {}^{10}C_0 + 17 . {}^{10}C_1 + 14 . {}^{10}C_2 + ...- 7 . {}^{10}C_9 - 10 . {}^{10}C_{10} =$
(1) $10.2^9 + 1$ (2) 9.2^{10} (3) 10.2^9 (4) none of these

18.14 $63.{}^{21}C_0 + 53.{}^{21}C_2 + 43.{}^{21}C_4 + ...+(-37).{}^{21}C_{20} =$
(1) 20.2^{20} (2) 21.2^{19} (3) 20.2^{19} (4) 19.2^{19}

18.15 $2.{}^nC_0 + 2^2 . \frac{{}^nC_1}{2} + 2^3.\frac{{}^nC_2}{3} + ... + 2^{n+1}.\frac{{}^nC_n}{n+1} =$
(1) $3^{n+1} - 1$ (2) $\frac{3^{n+1}-1}{n+1}$ (3) $\frac{3^{n+1}}{n+1}$ (4) $\frac{3^{n+1}-1}{n}$

18.16 If ${}^{11}C_r = C_r$, then $C_0 C_3 + C_1 . C_4 + ...+ C_8 . C_{11} =$
(1) ${}^{21}C_7$ (2) ${}^{22}C_8$ (3) ${}^{23}C_9$ (4) None of these

18.17 ${}^{100}C_{50} + {}^{99}C_{50} + {}^{98}C_{50} + ... + {}^{50}C_{50} =$
(1) ${}^{100}C_{50}$ (2) ${}^{100}C_{52}$ (3) ${}^{100}C_5$ (4) ${}^{101}C_{51}$

18.18 $\sum_{r=0}^{n-1}\left(\frac{{}^{n-1}C_r + {}^{n-1}C_{r-1}}{{}^nC_r + {}^nC_{r+1}}\right) =$
(1) $\frac{n}{2}$ (2) $\frac{n+1}{2}$ (3) $\frac{n(n+1)}{2}$ (4) $\frac{n(n-1)}{2(n+1)}$

18.19 The coeff. of x^4 in $(3 - 2x)^{-3/4}$; for $|x| < 1$, is
(1) $\frac{385}{128.3^4}$ (2) $\frac{385}{128.3^3}$ (3) $\frac{385}{128.3^3} \cdot 3^{-3/4}$ (4) $\frac{385}{64.3^3} \cdot 3^{-3/4}$

18.20 $1 + \frac{7}{24} + 2.\left(\frac{7}{24}\right)^2 + \frac{14}{3}\left(\frac{7}{24}\right)^3 + ... =$
(1) 2 (2) 3 (3) $\sqrt{2}$ (4) $\sqrt{3}$

18.21 The coefficient of $x^6 y^5 z^3$ in the expansion of $(3xy - 2xz + zy)^7$ is
(1) $\frac{-7!}{4!3!} \times 3^3.2^3$ (2) $\frac{7!}{4!\,2!\,1!} \times 3^4.2^2$ (3) $\frac{-7!}{4!\,2!\,1!}.3^4.2^2$ (4) None of these

18.22 The number of terms in $(3a + 2b + 4c)^{70}$ are
(1) ${}^{73}C_3$ (2) ${}^{72}C_2$ (3) ${}^{71}C_2$ (4) ${}^{72}C_3$

18.23 $\sum_{j=1}^{m}\sum_{i=1}^{n} i.j^2 =$

(1) $\frac{mn(n+1)(m+1)(2n+1)}{12}$
(2) $\frac{n(n+1)m(m+1)(2m+1)}{12}$
(3) $\frac{mn(n+1)(m+1)}{4}$
(4) none of these

18.24 If $(1 + 3x - 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ...a_{40}$, then the value of $a_1 + a_2 + a_3 + ... + a_{39}$ is
(1) – 1 (2) + 1 (3) 0 (4) 2^{20}

18.25 ${}^nC_1 \cdot 2 + {}^nC_2 \cdot \frac{2^2}{3} + {}^nC_3 \cdot \frac{2^3}{3^2} + ...{}^nC_n \cdot \frac{2^n}{3^{n-1}} =$

(1) $\frac{3^n - 2^n}{3^{n-1}}$ (2) $\frac{3^n + 2^n}{3^{n-1}}$ (3) $\frac{5^n - 3^n}{3^{n-1}}$ (4) $\frac{3^n + 5^n}{3^{n-1}}$

18.26 The greatest number from which $n(n^2 - 1)$ is divisible (where n is odd natural number)
(1) 48 (2) 16 (3) 12 (4) 24

18.27 Let $S(k) = 2 + 4 + 6 + ...+ 2k = -1 + k(k + 1)$, then which of the following is true
(1) Principle of mathematical induction can be used to prove formula
(2) $S(k) \Rightarrow S(k + 1)$
(3) $S(k) \not\Rightarrow S(k + 1)$
(4) S(1) correct

18.28 If $a_n = \sqrt{11 + \sqrt{11 + \sqrt{11 + ...}}}$ having n radical signs then by method of mathematical induction which is true
(1) $a_n > 11 \ \forall \ n \geq 1$ (2) $a_n < 11 \ \forall \ n \geq 1$ (3) $a_n < 4 \ \forall \ n \geq 1$ (4) $a_n < 3 \ \forall \ n \geq 1$

18.29 Let S(k) is the statement, $2.7^k + a.5^k - 5$ is divisible by 24 then the value of a for which S(k + 1) becomes true $\forall \ k \in N$
(1) 3 (2) 2 (3) 1 (4) 5

18.30 For all positive integer n, P(n) is the statement, $2^{n-2} > 3n$ then which of the following is true
(1) P(3) is true
(2) P(5) is true
(3) If P(m) is true then P(m + 1) is also true
(4) If P(m) is true then P(m + 1) is not true

18.31 $x^n - y^n$ is divisible by, $n \in N$
(1) x + y (2) x – y (3) both A and B (4) none of these

18.32 If S(n) is the statement, $10^{2n-a} + b$ is divisible by 11, then the value of a and b for which S(n + 1) is true
(1) a = 1, b = 3 (2) a = 3, b = 1 (3) a = 2, b = 4 (4) all of above

Level : II (Tough)

18.33 $\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} {}^{n+3}C_r =$

(1) $\frac{2^{n+5} - 2}{(n+4)(n+5)} - 1$
(2) $\frac{2^{n+5} - 2}{(n+4)(n+5)}$
(3) $\frac{2^{n+5} - 2}{(n+4)(n+5)} - \frac{2}{n+4} - 1$
(4) None of these

18.34 $^{10}C_0 \cdot {}^{20}C_{10} - {}^{10}C_1 \cdot {}^{18}C_{10} + {}^{10}C_2 \cdot {}^{16}C_{10} - {}^{10}C_3 \cdot {}^{14}C_{10} + \ldots\ldots = $

(1) 0 (2) 1 (3) 2^{10} (4) 3^{10}

18.35 $\sum_{m=0}^{n} \sum_{p=0}^{m} {}^{2n}C_{2m} \cdot {}^{2m}C_{2p} =$

(1) $\frac{3^{2n}+1}{4}$ (2) $\frac{3^{n}+1}{4}$ (3) $\frac{3^{2n}-1}{4}$ (4) None of these

18.36 If $\left(\frac{x-3}{-4}\right)^{5/7} = 1+\frac{5}{7}\left(\frac{x+1}{x-3}\right)+\frac{10}{49}\left(\frac{x+1}{x-3}\right)^2 + \ldots\ldots\infty$, then the set of values of x is

(1) $(-\infty, 1)$ (2) $(-\infty, 1) \cup (3, \infty)$ (3) R (4) R^-

18.37 Let S(n) is the statement, $11^{n+2} + 12^{2n+b}$ is divisible by 133 $\forall$ n $\in$ N, then which of the following is not the value of b. If S(n + 1) is also true

(1) 3 (2) 2 (3) 5 (4) None of these

18.38 If S(n) is the statement, $\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = \frac{(-1)^n n!}{x^{n+1}}\left(\log x - 1 - \frac{1}{2}\ldots\ldots\frac{-1}{n}\right)$, then which of the following is incorrect $\forall$ n $\in$ N

(1) above statement is true for n = 3
(2) above statement is true for n = 4
(3) If S(n) is true then S(n + 1) is also true
(4) None of these

18.39 For all positive integer n, S(n) is the statement, $\frac{n^7}{7}+\frac{n^5}{5}+\frac{2}{3}n^3 - \frac{n}{105}$ is an integer, then

(1) S(3) is not true
(2) S(5) is not true
(3) If S(k) is true then S(k + 1) is also true
(4) If S(k) is true then S(k + 1) is not true

SECTION - II : ASSERTION & REASONING TYPE

18.40 **Statement-1 :** The term independent of x in the expansion of $\left(x+\frac{1}{x}+2\right)^{21}$ is $^{42}C_{21}$

Statement-2 : In binomial expansion middle term is independent of x

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

18.41 **Statement-1 :** $2^{4n} - 2^n(7^n + 1)$ is divisible by square of 14, where x is a natural number

Statement-2 : $(1 + x)^n = 1 + {}^nC_1 x + \ldots\ldots\ldots + {}^nC_n \cdot x^n$ $\forall$ n $\in$ N, n $\geq$ 2

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 19 PERMUTATION & COMBINATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

19.1 At an election; a voter may vote for any number of candidates not greater than the number to be elected. There are 10 candidate and 4 are be elected. The number of ways in which a voter may vote for at least one candidate is -
(1) 385 (2) 1110 (3) 5040 (4) None of these

19.2 The sum of the digits in the unit place of all numbers formed with the help of 3, 4, 5, 6 taken all at a time is -
(1) 106 (2) 107 (3) 108 (4) None of these

19.3 Number of ways in which 9 different toys can be distributed among two brotihes in the ratio 1 : 2 is -
(1) 162 (2) 164 (3) 165 (4) 168

19.4 The number of order triplets of positive integers which are solutions of the equation $x+y+z = 100$ is
(1) 6005 (2) 4851 (3) 5081 (4) None of these

19.5 10 different letters of alphabet are given. Words with 5 letters are formed from these given letters. Then the number of words which have atleast one letter repeated is
(1) 69760 (2) 30240 (3) 99748 (4) None of these

19.6 Total number of words formed by using 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to
(1) 60 (2) 120 (3) 720 (4) none of these

19.7 If each of 10 points on a straight line be joined to each of 10 points on a parallel line then the total number of triangles that can be formed with the given points as vertices is
(1) 860 (2) 900 (3) 920 (4) none of these

19.8 The letter of the word RANDOM are written in all possible orders and these words are written out as in a dictionary. Then the rank of the word RANDOM is
(1) 614 (2) 615 (3) 613 (4) 616

19.9 If ${}^nC_{r-1} = 10$, ${}^nC_r = 45$ and ${}^nC_{r+1} = 120$, then r equals
(1) 1 (2) 2 (3) 3 (4) 4

19.10 Find the sum of all four digits numbers that can be formed with the digits 1, 2, 2 and 3
(1) 26664 (2) 206664 (3) 206644 (4) none of these

19.11 The number of seven letter words that can be formed by using the letters of the word SUCCESS so that the two C are together but no two S are together is :
(1) 36 (2) 24 (3) 54 (4) 48

19.12 The number of ways in which 10 boys and 10 girls can be seated in a row so that boys and girls alternate is
(1) $2 \times 10! \times 10!$ (2) $10! \times 10!$ (3) $2! \times 10!$ (4) $10! \times 11 \times 10!$

19.13 The number of divisors of 9600 including 1 and 9600 are
(1) 60 (2) 58 (3) 48 (4) 46

19.14 Four boys picked up 30 mangoes. the number of ways in which they can divide them if all mangoes be identical, is
(1) 5456 (2) 3456 (3) 5462 (4) none of these

19.15 The prime ministers of 9 countries meet to discuss the terrorism problem. The number of ways they can seat themselvs at a round table so that the Pakistan and India prime ministers sit together is (pearson)
(1) 2!.7! (2) 2.8! (3) 7! (4) 8!

19.16 The maximum no. of points in which 5 circles and 4 straight lines intersect is
(1) 66 (2) 60 (3) 33 (4) 46

19.17 In a shelf there are 5 physics, 4 mathematics and 3 chemistry books. How many combinations are there if books of same subject are different
(1) 4095 (2) 8191 (3) 139 (4) 140

19.18 There are 12 Balls in a basket of which 5 are red 4 black and 3 blue balls blue balls are different. How many ways are there to select atleast one ball.
(1) 239 (2) 119 (3) 240 (4) 120

19.19 In how many ways three bundle of 5 books be distributed in 4 students
(1) 150 (2) 600 (3) 200 (4) 800

19.20 In how many ways can 5 rings be worn on four finger if any no. of rings can be worn on any finger:
(1) 625 (2) 1024 (3) 125 (4) 512

19.21 There are four different letters be posted in different envelopes in how many ways they can be put to these envelopes one in each so that no letter goes to the correct envelope
(1) 9 (2) 12 (3) 11 (4) 13

19.22 Two straight lines intersect at a point R. Points P_1, P_2.......P_n are taken on one line and points Q_1, Q_2.....Q_n on the other. If the point R is not to be used, the number of triangles that can be drawn using these points as vertices, is
(1) $n(n-1)$ (2) $n^2(n-1)^2$ (3) $n(n-2)^2$ (4) $n^2(n-1)$

19.23 If 44! is divisible by 3^n then maximum value of n equal to
(1) 20 (2) 19 (3) 21 (4) 18

19.24 Three boys and three girls are to be seated around a table, in a circle. Among them the boy x does not want any girl neighbour and the girl y does not want any boy neighbour. The number of such arrangement is
(1) 4 (2) 6 (3) 8 (4) 10

Level : II (Tough)

19.25 A train going from london to cambridge stops at 12 intermediate stations. 75 persons enter the train with 75 different tickets of the same class. Number of different sets of tickets they may be holding is :
(1) ${}^{78}C_3$ (2) ${}^{91}C_{75}$ (3) ${}^{84}C_{75}$ (4) None

19.26 In a unique hockey series between india and pakistan, they dicide to play on till a team wins 5 matches. The number of ways in which the series can be won by india if no match ends in a draw is :
(1) 126 (2) 252 (3) 225 (4) 276

19.27 Six persons A, B, C, D, E and F are to be seated at a circular table . The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right is
(1) 36 (2) 12 (3) 18 (4) 24

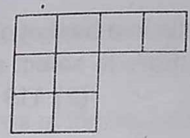
19.28 The number of permutations which can be formed out of the letters of the word "SERIES" taking three letters together, is:
(1) 120 (2) 60 (3) 42 (4) none

19.29 The number of 7 digit numbers with sum of digits equal to 10 and formed by using the digits 1, 2 and 3 only.
(1) 55 (2) 66 (3) 74 (4) 188

19.30 No. of ways of arranging exactly 4 fruits from 5 apples, 4 mangoes and 2 oranges.
(1) 12 (2) 14 (3) 17 (4) 15

19.31 Number of rectangle excluding square in chess board is
(1) 204 (2) 1092 (3) 1296 (4) 672

19.32 The number of different ways the letters of the word ORANGE can be placed in the 8 boxes of the given below such that no row remains empty, is equal to



(1) 26
(2) 26 × 6!
(3) 6!
(4) 2! × 6!

19.33 There are 8 offical and 4 non-official members and out of these 12 members, in how many ways a committee of 5 is to be formed such that 3 official and 2 non-official are in committee
(1) 363 (2) 336 (3) 236 (4) 326

SECTION - II : ASSERTION & REASONING TYPE

19.34 **Statement-1 :** If N is the number of positive integral solutions of $x_1x_2x_3x_4 = 880$, then N is divisible by 5 distinct primes.
Statement-2 : Prime numbers are 2,3,5,7,11,13.....
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

19.35 **Statement-1 :** The number of ways in which 30 identical things can be distributed among 8 persons if each person gets atleast 2 things is ${}^{21}C_7$
Statement-2 : Coeff. of x^r in $(1 - x)^{-n}$ is ${}^{n+r-1}C_r$
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

19.36 **Statement-1 :** The number of non negative integral solutions of $x + y + z \le n$, where $n \in N$ is ${}^{n+3}C_2$
Statement-2 : Inequation of the form $x_1 + x_2 + x_3.....+x_m \le n$ can be solved by introducing a dummy. Variable x_{m+1} such that $x_1 + x_2......+x_m + x_{m+1} = n$, where $x_{m+1} \ge 0$
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

19.37 **Statement-1 :** $\frac{n+1!}{n-1!}$ is divisible by 6 for some $n \in N$

Statement-1 : Product of 3 consecutive integers is divisible by 3!

(1) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true ; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

19.38 **Statement-1 :** The maximum number of point of intersection of 8 unequal circles is 56.

Statement-1 : The maximum number of point into which 4 unequal circle and 4 non coincident straight lines intersect is 50.

(1) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true ; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

19.39 **Statement-1 :** If a, b, c are positive integers such the $a + b + c \leq 8$, then the number of posible value of the ordered triplets (a, b, c) is 56.

Statement-1 : The number of ways in which n identical things can be distributed into r different groups is ${}^{n-1}C_{r-1}$

(1) Statement-1 is true, statement-2 is true ; statement-2 is correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true ; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

TOPIC 20

PROBABILITY

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

20.1 A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. If p denotes the probability that 7 comes before 5, then the value of 5p is

(1) 1 (2) 2 (3) 3 (4) 4

20.2 In bag A there are 5 white and 3 black balls. In bag B there are 3 white and 1 black balls. One ball is chosen at random from any bag and found to be white then the probability that it is from bag B is

(1) $\frac{5}{11}$ (2) $\frac{6}{11}$ (3) $\frac{3}{8}$ (4) none of these

20.3 The probability that the birth days of six different persons will fall in exactly two calendar months, is -

(1) $\frac{341}{12^5}$ (2) $\frac{66}{12^5}$ (3) $\frac{352}{12^5}$ (4) None of these

20.4 A and B are two matrices with 32 and 56 elements respectively then the probability that (A×B) is possible, is-

(1) $\frac{1}{9}$ (2) $\frac{1}{11}$ (3) $\frac{1}{12}$ (4) $\frac{1}{14}$

20.5 Let X be a set containing n elements. If two subsets A and B of X are picked at random, the probability that A and B have the same number of elements is -

(1) $\frac{^{2n}C_n}{2^n}$ (2) $\frac{1}{^{2n}C_n}$ (3) $\frac{1.3.5......(2n-1)}{2^n\,n!}$ (4) $\frac{3^n}{4^n}$

20.6 A four digit number is formed by the digits 1, 2, 3, 4, 5, 6 and 8. The probability that the number has odd digits at both ends is :

(1) $\frac{2}{7}$ (2) $\frac{3}{7}$ (3) $\frac{1}{7}$ (4) none of these

20.7 Five digits from the numbers 1, 2, 3, 4, 5, 6, 7 are written in random order. If the probability that this 5-digit number is divisible by 4 is λ then the value of 28 λ is

(1) 5 (2) 6 (3) 7 (4) 8

20.8 The letters of the word RESONANCE arranged at random. The probability that the vowels may occupy the even places, is

(1) $\frac{3}{126}$ (2) $\frac{1}{126}$ (3) $\frac{5}{126}$ (4) $\frac{1}{64}$

20.9 A coin is tossed and a die is thrown. Let 'A' be the event 'T turn's up on the coins and odd number turns up on the die' and B be the event 'H turns up on the coin and an even number turns up on the die' write the probability A and B, A or B respectively

(1) 0, $\frac{1}{2}$ (2) 0, $\frac{1}{3}$ (3) $\frac{1}{2}$, 0 (4) $\frac{1}{3}$, 0

20.10 If A and B are events such that P(A) = 0.42, P(B) = 0.48 and $P(A \cap B) = 0.16$, then $P(\overline{A+B})$ is equal to

(1) 0.90 (2) 0.46 (3) 0.26 (4) 0.54

20.11 The probability that 13th day of randomly selected month is a second saturday is

(1) $\frac{3}{84}$ (2) $\frac{5}{84}$ (3) $\frac{1}{84}$ (4) $\frac{13}{84}$

20.12 In throwing a pair of dies getting an even number on first die and a total of 7 on both the dies is

(1) 1/6 (2) 1/12 (3) 1/4 (4) 5/12

20.13 If A and B are two events such that P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{4}$ and $P(A \cap B) = \frac{1}{5}$

then $P(\overline{B}/\overline{A})$ =

(1) 3/40 (2) 13/40 (3) 27/40 (4) 37/40

20.14 If the odds in favour of an event be $\frac{4}{5}$, then the probability of non occurance of the event is

(1) 8/9 (2) 4/9 (3) 5/9 (4) none of these

20.15 The probability distribution of a random variable X is given below. Then its mean is

$x = x_i$	1	2	3	4
$P(x = x_i)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

(1) 2 (2) 1 (3) 3 (4) 4

20.16 A purse contain 4, 10 paise coin; 3, 25 paise coin; 2, 50 paise coin. A coin is drawn at random. Find the probability of coin drawn is 25 paise or 50 paise is -

(1) $\frac{1}{2}$ (2) $\frac{5}{9}$ (3) $\frac{4}{9}$ (4) $\frac{1}{3}$

20.17 There are 4 white and 5 black ball in a Box. In an another box there are 5 white and 4 black balls. An unbiased die is rolled. If it shows even no. then a ball is drawn from the second box otherwise from first box. If the ball drawn is white then the probability that the ball was drawn from the first box is

(1) $\frac{5}{9}$ (2) $\frac{4}{9}$ (3) $\frac{7}{9}$ (4) $\frac{1}{9}$

20.18 In a box containing 100 eggs, 20 eggs are rotten. The probability that out of a sample of 5 eggs none is rooten. If the sampling is with replacement is

(1) $\left(\frac{3}{5}\right)^5$ (2) $\left(\frac{4}{5}\right)^5$ (3) $\left(\frac{2}{5}\right)^5$ (4) $\left(\frac{1}{10}\right)^5$

20.19 Three players A, B, C in this order, Draw a card from well shuffled pack of card and reshuffled after each draw. If the winner is one who draws a red card, then C's chance of winning is

(1) $\frac{1}{8}$ (2) $\frac{1}{7}$ (3) $\frac{1}{6}$ (4) $\frac{1}{5}$

20.20 There are 3 bags which are known to contain 2 white and 3 black balls, 4 white and 2 black balls and 3 white and 2 black balles. A bag is drawn randomly from one of the bag find the probability of ball being black.

(1) $\frac{7}{9}$ (2) $\frac{5}{9}$ (3) $\frac{4}{9}$ (4) $\frac{1}{9}$

20.21 The probability of a man hitting a target is $\frac{2}{3}$. He tries 5 times. The probability that the target will be hit at least 3 times, is

(1) ${}^5C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$ (2) ${}^5C_4\left(\frac{2}{3}\right)^4\left(\frac{2}{3}\right)^1$ (3) ${}^5C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^0$ (4) none of these

20.22 In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to questions is 75%. If he gets the correct answer to a question, the probability that he was guessing is

(1) $\frac{1}{13}$ (2) $\frac{1}{4}$ (3) $\frac{2}{13}$ (4) $\frac{7}{3}$

20.23 In 8 trials of an experiment, if the probability of getting '3 success' is maximum, then the probability of failure in each trial is -

(1) $\frac{3}{8}$ (2) $\frac{5}{8}$ (3) $\frac{1}{8}$ (4) $\frac{1}{4}$

20.24 In an entrance test there are multiple choice questions. There are four passible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90% then the probability of correct answer is -

(1) $\frac{37}{40}$ (2) $\frac{41}{47}$ (3) $\frac{1}{3}$ (4) $\frac{33}{40}$

Level : II (Tough)

20.25 A purse contain five coins each of which is either a rupee or two rupees coins. Find the expected value of a coin in that purse

(1) 6.5 (2) 17.5 (3) 7.5 (4) 8.5

20.26 E and F be two independent events such that P(E) < P(F). The probabilility that both E and F happen is 1/12 and the probability that niether E nor F happen is 1/2. Then

(1) P(E) = 1/3 , P(F) = 1/2 (2) P(E) = 1/2 , P(F) = 2/3
(3) P(E) = 2/3 , P(F) = 3/4 (4) P(E) = 1/4 , P(F) = 1/3

20.27 A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six is :–

(1) 3/8 (2) 1/5 (3) 3/4 (4) None of these

20.28 Three persons A,B &C are to speak at a function along with 5 others. If the persons speaks in a random order The probability that A speaks before B & B speaks before C is

(1) $\frac{3}{8}$ (2) $\frac{1}{12}$ (3) $\frac{1}{8}$ (4) $\frac{1}{6}$

20.29 If m is a natural number such that m ≤ 5, then the probability that quadratic eqaution $x^2 + mx + \frac{1}{2} + \frac{m}{2} = 0$ has real roots is

(1) $\frac{1}{5}$ (2) $\frac{2}{3}$ (3) $\frac{3}{5}$ (4) $\frac{1}{5}$

SECTION - II : ASSERTION & REASONING TYPE

20.30 **Statement-1 :** Out of 7 tickets consecutively numbered three are drawn at random, the chance that the numbers on them are in A.P. is $\frac{9}{35}$

Statement-2 : Out of (2n – 1) tickets consecutively numbered three are drawn at random, the chance that the numbers on them are in A.P. is $\frac{2n+3}{(2n-1)(2n-3)}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

20.31 **Statement-1 :** If A and B are two events in a sample space such that P(A) = .3, P(B) = .3, then $P(A \cap B)$ can not be found

Statement-2 : $P(\overline{A} \cap B) = P(B) - P(A \cap B)$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

20.32 Let A , B , C be three mutually independent events.

Statement–1 : A & $B \cup C$ are independent

Statement–2 : A & $B \cap C$ are independent

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

20.33 Let A & B be two events such that $P(A \cup B) = P(A \cap B)$.

Statement–1 : $P(A \cap B') = P(A' \cap B) = 0$

Statement–2 : P(A) + P(B) = 1

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

20.34 **Statement–1** : The probability of drawing either a ace or a king from a pack of card in a single draw is $\frac{2}{13}$

Statement–2 : For two events E_1 & E_2 which are not mutually exclusive, probability is given by $P(E_1 \cup E_2) = PE_1) + P(E_2) - P(E_1 \cap E_2)$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 21 MATRICES & DETERMINANT

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

21.1 If $\Delta(x) = \begin{vmatrix} x^2+4x-3 & 2x+4 & 1 \\ 2x^2+5x-9 & 4x+5 & 2 \\ 8x^2-6x+1 & 16x-6 & 8 \end{vmatrix} = ax^3 + bx^2 - cx + d$, then

(1) $a = 0$ (2) $b = 0$ (3) $c = 0$ (4) $d = 189$

21.2 Matrix A is given by $A = \begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix}$, then the determinant of $(A^{2005} - 6.A^{2004})$, is -

(1) 2^{2006} (2) $(-11).\ 2^{2005}$ (3) -2^{2005} (4) $(-9).\ (2)^{2004}$

21.3 If 3 digit numbers A28, 3B9 and 62C are divisible by a fixed constant 'K' where A, B, C are integers lying between 0 and 9, then determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is always divisible by

(1) K (2) A (3) ABC (4) K^2

21.4 If $0 \le [x] < 2$; $-1 \le [y] < 1$ and $1 \le [z] < 3$ where [.] denotes the greatest integer function, then the maximum value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is -

(1) 2 (2) 4 (3) 6 (4) 8

21.5 If $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = 0$ and a, b, c are different real numbers, then the value of $3(a + b + c) + 2$ is

(1) 5 (2) 0 (3) 2 (4) 6

21.6 If the system of equation

$\lambda p + q + r = 0$

$p + \lambda q + r = 0$

$p + q + \lambda r = 0$

has non trivial solution, then the value of λ can be the roots of quadratic equation, which is

(1) $x^2 + x - 2 = 0$ (2) $x^2 - x + 2 = 0$ (3) $x^2 + 4x + 1 = 0$ (4) $x^2 - 3x + 2 = 0$

21.7 If $f(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, $g(x) = \int_0^x f(t)\,dt$, then range of $g'(x)$ equal to

(1) $\left[0, \frac{1}{2}\right]$ (2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (3) $(0, \infty)$ (4) none of these

21.8 If $\Delta_r = \begin{vmatrix} r.r! & {}^nC_r & 2r \\ (n+1)! & 2^n & n^2+n+2 \\ 1 & 1 & 2 \end{vmatrix}$, then $\sum_{r=1}^{n} \Delta_r$ is equal to

(1) 0 (2) $\frac{n(n+1)}{2}$ (3) n^2 (4) $\sum_{r=2}^{n} r$

21.9 If $\Delta_r = \begin{vmatrix} r & n & 6 \\ r^2 & 2n^2 & 4n-2 \\ r^3 & 3n^3 & 3n^2-3n \end{vmatrix}$, then $\sum_{r=0}^{n-1} \Delta_r$ equals to

(1) $n^2(n+2)$ (2) $n(n+2)^2$ (3) $\frac{1}{12}$ $n(n^3+2)$ (4) none of these

21.10 If $\Delta = \begin{vmatrix} 2 & 2 & 2 \\ {}^nC_1 & {}^{n+3}C_1 & {}^{n+6}C_1 \\ {}^nC_2 & {}^{n+3}C_2 & {}^{n+6}C_2 \end{vmatrix}$ = x, then number of possible relative prime factors of 'x', is

(1) 0 (2) 2 (3) 4 (4) none of these

21.11 If A and B are two 3 × 3 order matrices, then which one of the following is not true.
(1) $(A + B) = A' + B'$ (2) $(AB)' = A'\,B'$
(3) det (AB) = det (A) det B (4) $A(\text{adj}\,A) = |A|I_3$

21.12 If $A = \begin{bmatrix} x & 3 & 2 \\ -3 & y & -7 \\ -2 & 7 & 0 \end{bmatrix}$ and $A = -A'$, then x + y is equal to

(1) 2 (2) – 1 (3) 0 (4) 12

21.13 If A is 3 × 3 skew symmetric matrix, then trace of A is equal to
(1) 1 (2) |A| (3) – 1 (4) none of these

21.14 The number of values of k for which the system of equation (k + 1) x + 8y = 4k, kx + (k + 3) y = 3k – 1 has no solution is
(1) 3 (2) 1 (3) 2 (4) infinite

21.15 The system of equations
$(p\alpha + q)\,x + py + qz = 0$
$(q\alpha + r)\,x + qy + rz = 0$
$(p\alpha + q)\,y + (q\alpha + r)\,z = 0$
has a non trivial solution if

(1) 2p = q + r (2) $\frac{p}{r} = \left(\frac{q}{p}\right)^2$ (3) $\frac{2}{q} = \frac{1}{p} + \frac{1}{r}$ (4) $p\alpha^2 + 2q\alpha + r = 0$

21.16 If a matrix A satisfy $A^2 - 5A + 7I = 0$ and $A^8 = pA + qI$ then the value of p is -
(1) 9621 (2) 1265 (3) 5299 (4) undefine

21.17 If A is a skew-symmetric matrix such that $A^2 + I = 0$, then
(1) A is nilpotent matrix of even order
(2) A is orthogonal matrix of even order
(3) A is involutory matrix of odd order
(4) A is singular matrix

21.18 If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $f(-\alpha)$ is equal to

(1) $f^{-1}(-\alpha)$ (2) $[f(\alpha)]^{-1}$ (3) $-f(\alpha)$ (4) none of these

21.19 If p is a constant and $f(x) = \begin{vmatrix} x^2 & x^3 & x^4 \\ 2 & 3 & 6 \\ p & p^2 & p^3 \end{vmatrix}$ if $f''(x) = 0$ have roots α, β, then

(1) α and β have opposite sign and equal magnitude at $p = \sqrt{3}$
(2) At p = 1, $f''(x) = 0$ represent an identity
(3) At p = 2, product of roots are unity
(4) At $p = -\sqrt{3}$ product of roots are positive

21.20 If $\Delta_r = \begin{vmatrix} a^r & b^r & c^r \\ a & b & c \\ 1-a & 1-b & 1-c \end{vmatrix}$. If $\sum_{r=0}^{\infty} \Delta_r = 0$, then

which statement is true.

(1) $a = b = c, a \in R$ (2) $a = b = c, |a| < 1$ (3) $a = b = c \neq 1$ (4) $a = b \neq c, |c| < 1$

21.21 If the matrix $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is an orthogonal matrix, then

(1) $c = \frac{1}{\sqrt{3}}$ (2) $a = \frac{1}{\sqrt{6}}$ (3) $b = -\frac{1}{\sqrt{2}}$ (4) $c = -\frac{1}{\sqrt{6}}$

21.22 If AB = A and BA = B, then

(1) $A^2 = B$ (2) $A^2 = A$ (3) $B^2 = 2B$ (4) $B^2 = A$

Level : II (Tough)

21.23 The value of $\Delta = \begin{vmatrix} {}^{16}C_8 & {}^{16}C_8 & {}^{15}C_8 \\ {}^{14}C_7 & {}^{14}C_6 & {}^{13}C_5 \\ {}^{12}C_6 & {}^{12}C_5 & {}^{11}C_4 \end{vmatrix}$ is

(1) 25 (2) 52 (3) ${}^{14}C_8 - {}^{13}C_5$ (4) $2 \cdot {}^{13}C_6 - {}^{14}C_7$

21.24 If $a + b + c \neq 0$, then system of equation

$(b + c)(y + z) - ax = b - c$
$(c + a)(z + x) - by = c - a$
$(a + b)(x + y) - cz = a - b$

has

(1) a unique solution (2) no solution
(3) infinite number of solution (4) exactly two solution

21.25 If $\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix}\begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then the number of values of 'θ' in $[-2\pi, 2\pi]$ satisfying it, is

(1) 0 (2) 3 (3) 5 (4) 7

21.26 If the rank of the matrix $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & 7 \\ 5 & 8 & 8 \end{bmatrix}$ is 'α', then find the value of $\int_0^{\sqrt{\alpha}} [x^2]\,dx$, where [.] denote greatest integer function.

(1) $\sqrt{8}-\sqrt{2}-1$ (2) $\sqrt{9}$ (3) $\sqrt{12}-\sqrt{2}-1$ (4) $\sqrt{9}-\sqrt{2}+1$

21.27 If $f(\sin x) = \begin{vmatrix} \sin x & 2\cos x & 2\tan x \\ 1 & \sin x & \cos x \\ \tan x & \cos x & \sin x \end{vmatrix}$ and $g(x) = \int f(x)dx$, $g(\sqrt{2}) = 0$, then

(1) g(x) is an odd function (2) $\lim\limits_{x\to 1} f(x) = 2$

(3) $g(2) = 9 + \ell n\, 3$ (4) Time period of f(sin x) is π

21.28 $\Delta(x) = \begin{vmatrix} \tan x & \sin x & \cos x \\ \sqrt{2} & 1 & 1 \\ 2 & \sqrt{3} & 3 \end{vmatrix}$, then $\Delta'(x) = 0$ has

(1) no solution in $\left(0, \frac{\pi}{4}\right)$ (2) Atleast two solution in $\left[0, \frac{\pi}{2}\right)$

(3) Atleast one solution in $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ (4) No solution in $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

21.29 Let P denotes the set of all values of λ for which the system of equation

$$\lambda x_1 + x_2 + x_3 = 1$$
$$x_1 + \lambda x_2 + x_3 = 1$$
$$x_1 + x_2 + \lambda x_3 = 1$$

is inconsistent, then

(1) $\sum\limits_{\lambda \in P} |\lambda| = 2$ (2) λ is an even prime number

(3) $\ell im_{x\to\lambda} \frac{|x+2|}{x^2-4} = \frac{1}{4}$ (4) Cube roots of λ are 1, ω, ω^2

21.30 If α, β, γ are the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$, then find the determinant $\Delta = \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

(1) $\frac{b^3-3abc}{a^3}$ (2) $\frac{b^2-4ac}{a^2}$ (3) 0 (4) $\frac{b^3-4abc}{a^3}$

SECTION - II : ASSERTION & REASONING TYPE

21.31 **Statement-1 :** $\begin{vmatrix} i^{99} & 0 & 0 \\ -\omega^{20} & i^{98} & 0 \\ \omega & 20 & 2i^{97} \end{vmatrix} = -2$, where ω and i are cube and fourth root of unity respectively.

Statement-2 : If all the diagonal element of a determinant are zero and $a_{ij} = -a_{ji}$, then its value is always zero.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

21.32 For a system of equation AX = B

Statement-1 : System have unique solution if B is a non singular matrix and matrix A can be singular.

Statement-2 : Singular matrix have value of its determinant equal to zero.

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

21.33 **Statement-1 :** $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} a & b & c \\ b & c+1 & a-1 \\ c & a-1 & b+1 \end{bmatrix}$, then AB = BA is possible if $B = A^{-1}$ or value of a + b + c = 1.

Statement-2 : AI = IA is possible when I is the unit matrix of order '3' or $I = A^{-1}$

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

21.34 **Statement-1 :** $\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} = 0$

Statement-2 : $\log_b a = \dfrac{\log a}{\log b}$ and if any two rows are identical then $\Delta = 0$

(1) Statement -1 is True, Statement -2 is True ; Statement -2 is a correct explanation for Statement -1
(2) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(3) Statement -1 is True, Statement -2 is False
(4) Statement -1 is False, Statement -2 is True

21.35 **Statement 1 :** If $A = \begin{bmatrix} a^2+x^2 & ab-cx & ac+bx \\ ab+xc & b^2+x^2 & bc-ax \\ ac-bx & bc+ax & c^2+x^2 \end{bmatrix}$ and $B = \begin{bmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{bmatrix}$, then $|A| = |B|^2$.

Statement 2 : If A^c is cofactor matrix of a square matrix A of order n then $|A^c| = |A|^{n-1}$.
(1) Statement-1 is true, statement-2 is true; statment-2 is a correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(3) Statement-1 is true, statement- is false.
(4) Statement-1 is false, statement-2 is true.

TOPIC 22 COMPLEX NUMBER

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

22.1 If $\frac{z+1}{z+i}$ is purely Imaginary, then z lies on a -
(1) straight line (2) circle
(3) Circle with radius 1 (4) circle passing through (1, 1).

22.2 One vertex of square is 1 – i. Intersection point of diagonal is at origin. Then extremities of diagonal not passing through given vertex are -
(1) 1 + i (2) 1 – i (3) –1 + i (4) None of these

22.3 If $(x - iy) + i(3x + iy) = \frac{i^{4n+3} + (-i)^{8n-3}}{(-i)^{12n-1} - i^{2-16n}}$, $n \in N$ then pair (x, y) is -
(1) (0, –1) (2) (1, 2) (3) (0, 1) (4) (–1, –2)

22.4 If 1, α_1, α_2, α_3, α_4, α_5, α_6, are seven, 7th root of unity then $|(3 - \alpha_1)(3 - \alpha_3)(3 - \alpha_5)|$ is -
(1) $\sqrt{2186}$ (2) $\sqrt{1093}$ (3) $\sqrt{1023}$ (4) $\sqrt{511}$

22.5 If 1, α_1, α_2, ... α_{n-1} are nth root of unity then $\frac{1}{2-\alpha_1} + \frac{1}{2-\alpha_2} + \text{..........} + \frac{1}{2-\alpha_{n-1}}$ is -
(1) $\frac{(n-2)2^n+1}{2^n-1}$ (2) $\frac{(n-2)2^n,-1}{2^n-1}$ (3) $\frac{(n-2)2^{n-1}-1}{2^n-1}$ (4) $\frac{(n-2)2^{n-1}+1}{2^n-1}$

22.6 If $(x - iy) + i(3x + iy) = \frac{i^{4n+3} + (-i)^{8n-3}}{(-i)^{12n-1} - i^{2-16n}}$, $n \in N$, then find pair (x, y) -
(1) (0, –1) (2) (1, 2) (3) (0, 1) (4) (–1 –2)

22.7 $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be purely real if θ =
(1) $n\pi \pm \frac{\pi}{3}$, $n \in I$ (2) $n\pi$, $n \in I$ (3) $2n\pi \pm \frac{\pi}{3}$, $n \in I$ (4) none of these

22.8 Amplitude of $\sin\frac{\pi}{5} + i\left(1-\cos\frac{\pi}{5}\right)$
(1) $\frac{\pi}{5}$ (2) $\frac{2\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $\frac{\pi}{15}$

22.9 If $\sqrt{-7-24i} = a + ib$, then find value of $a^3 + b^3$ –
(1) 91 (2) 37 (3) – 91 (4) none of these

22.10 If $|z| \le 2$, then maximum & minimum value of $|z-4|$ are –
(1) 6, 0 (2) 6, –2 (3) 4, 2 (4) 6, 2

22.11 If z is a complex number such that $|3z-2| = |3z-4|$ then locus of z is –
(1) Circle (2) Straight line (3) Point (4) Ellipse

22.12 z_1, z_2, z_3 are three vertices of an equilateral triangle circumscribing the circle $|z| = \frac{1}{2}$. If $z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ and z_1, z_2, z_3 are in anticlockwise sense then z_2 is-
(1) $1 + \sqrt{3}i$ (2) $1 - \sqrt{3}i$ (3) 1 (4*) – 1

22.13 If conjugate of $(x + iy)(1 - 2i)$ be $1 + i$ then x and y are
(1) $\frac{3}{5}, \frac{1}{5}$ (2) $-\frac{1}{5}, -\frac{7}{5}$ (3) $\frac{3}{5}, \frac{-7}{5}$ (4) $-\frac{1}{5}, \frac{1}{5}$

22.14 Roots of the equation $z^n = (z+1)^n$ on the complex plane lie on the line -
(1) $2x + 1 = 0$ (2) $2x - 1 = 0$ (3) $x + 1 = 0$ (4) $x - 1 = 0$

22.15 Sum of common roots of $z^{2006} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$ is
(1) 0 (2) – 1 (3) 1 (4) 2

22.16 If $\omega \ne 1$ is a cube root of unity, then sum of series $S = 1 + 2\omega + 3\omega^2 + \ldots\ldots + 3n\omega^{3n-1}$ $(n \in N)$ is -
(1) $\frac{n}{\omega - 1}$ (2) $3n(\omega^2 - 1)$ (3) 0 (4) None of these

22.17 If $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$, ω is cube root of unity then value of $x^3 + y^3 + z^3$
(1) $a^3 + b^3$ (2) $3(a^3 + b^3)$ (3) $a^3 - b^3$ (4) $3(a^3 - b^3)$

22.18 If $z = \cos\theta + i\sin\theta$, then $\frac{z^{2n}-1}{z^{2n}+1}$ is equal to-
(1) $i\cot n\theta$ (2) $-i\cot n\theta$ (3) $i\tan n\theta$ (4) $-i\tan n\theta$

22.19 If z_1, z_2 are two complex numbers such that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then $\frac{z_1}{z_2}$ is -
(1) zero (2) purely real (3) purely imaginary (4) None of these

22.20 Equation $z\bar{z} + (2-3i)z + (2+3i)\bar{z} + 4 = 0$ represent a circle of radius –
(1) 3 (2) $\sqrt{13}$ (3) 2 (4) None of these

22.21 If $z = re^{i\theta}$ then $\arg(e^{iz})$ is –
(1) $-r\sin\theta$ (2) $r\cos\theta$ (3) $e^{-r\sin\theta}$ (4) $-r\cos\theta$

22.22 If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are seven, 7th root of unity (taken in counter clock wise sequence) then $|(3-\alpha_1)(3-\alpha_3)(3-\alpha_5)|$ is equal to
(1) $\sqrt{2186}$ (2) $\sqrt{1093}$ (3) $\sqrt{1023}$ (4) $\sqrt{511}$

22.23 Complex number z satisfying the inequality $|z - 5i| \le 3$ having least positive argument is-
(1) $\frac{12-16i}{5}$ (2) $\frac{16-12i}{5}$ (3) $\frac{16+12i}{5}$ (4) $\frac{12+16i}{5}$

22.24 $\frac{z-(1+i)}{z+(1+i)}$ is pure imaginary then z lies on

(1) a line segment (2) a circle (3) straight line (4) none of these

22.25 Vector z = 3 – 4i is turned anticlockwise through an angle 180° and stretched 2.5 times. Complex number corresponding to newly obtained vector is -

(1) $\frac{15}{2} - 10i$ (2) $-\frac{15}{2} + 10i$ (3) $-\frac{15}{2} - 10i$ (4) None of these

Level : II (Tough)

22.26 If origin and non-real roots of $2z^2 + 2z + \lambda = 0$ form three vertices of an equilateral triangle in argand plane then λ is –

(1) 2 (2) $\frac{2}{3}$ (3) – 1 (4) $\frac{3}{2}$

22.27 If $1, \alpha_1, \alpha_2 .... \alpha_{n-1}$ are n, n^{th} roots of unity then find

$$\frac{1}{2-\alpha_1} + \frac{1}{2-\alpha_2} + ... + \frac{1}{2-\alpha_{n-1}}$$

(1) $\frac{(n-2)2^n+1}{2^n-1}$ (2) $\frac{(n-2)2^n-1}{2^n-1}$ (3) $\frac{(n-2)2^{n-1}-1}{2^n-1}$ (4) $\frac{(n-2)2^{n-1}+1}{2^n-1}$

22.28 If $iz^3 + z^2 - z + i = 0$, then |z| equal to -

(1) 4 (2) 3 (3) 2 (4) 1

22.29 If n is an integer, then

$(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n$ is euqal to

(1) $2^n \cos^n \frac{\theta}{2} . \cos\frac{n\theta}{2}$ (2) $2^n \cos^n \frac{\theta}{2} . \sin\frac{n\theta}{2}$

(3) $2^{n+1} . \cos^n \frac{\theta}{2} . \sin\frac{n\theta}{2}$ (4) $2^{n+1} . \cos^n \frac{\theta}{2} . \cos\frac{n\theta}{2}$

22.30 Find the value of $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$, here ω is complex cube root of unity.

(1) 1 (2) – 1 (3) – 2 (4) 3

22.31 If $|z-1| + |z+3| \le 8$ then find the range of values of $|z-4|$

(1) $(1, \infty)$ (2) (1, 2) (3) [1, 9] (4) (0, 3)

22.32 If x,y are real and $-3 + x^2yi$, $x^2 + y + 4i$ are conjugate of each other, then |x| + |y| is equal to -

(1) 2 (2) 5 (3) 4 (4) 7

22.33 $1, \alpha, \alpha^2, ... \alpha^{n-1}$ are n, n^{th} roots of unity and $x^n - 1 = (x-1)(x-\alpha)(x-\alpha^2)......(x-\alpha^{n-1})$, then $(1-\alpha)(1-\alpha^2)(1-\alpha^3)......(1-\alpha^{n-1})$ is

(1) n –1 (2) n (3) 0 (4) not defined

SECTION - II : ASSERTION & REASONING TYPE

22.34 **Statement-1 :** $\left|\frac{3z+i}{2z+3+4i}\right| = 1.5$ represents a straight line

Statement-2 : Perpedicular bisector of a segment is a straight line

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

22.35 **Statement-1 :** Minimum of $f(\theta) = \left|\frac{2i}{3 - ie^{i\theta}}\right|$ is $\frac{1}{\sqrt{2}}$

Statement-2 : Maximum value of $f(\theta) = \left|\frac{2i}{3 - ie^{i\theta}}\right|$ is 1

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

22.36 **Statement-1 :** Centre of circle $\frac{|z+1|}{|z-1|} = 2$ is $\left(\frac{5}{3}, 0\right)$

Statement-2 : radius of circle $\frac{|z+1|}{|z-1|} = 2$ is $\frac{4}{3}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

22.37 **Statement-1 :** If z_1, z_2 , z_3 are such that $|z_1| = |z_2| = |z_3|$ 1, then maximum value of $|z_2 - z_3|^2 + |z_3 - z_1|^2 + |z_1 - z_2|^2$ is 9

Statement-2 : If z_1, z_2, z_3 are such that $|z_1| = |z_2| = |z_3| = 1$, then is $(z_2\bar{z}_3 + z_3\bar{z}_1 + z_1\bar{z}_2) \geq -\frac{3}{2}$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

22.38 **Statement-1 :** If $\omega \neq 1$ is a cube root of unity and z is a complex number such that $|z| = 1$, then

$$\left|\frac{2 + 3\omega + 4z\omega^2}{4\omega + 3\omega^2 z + 2z}\right| = 1$$

Statement-2 : If z_1, z_2 are two complex numbers then $|z_1| = |z_2| \Rightarrow z_1 = \bar{z}_2$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 23 VECTOR

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

23.1 Two vectors $\vec{a} = 3\hat{i}+4\hat{j}$ and $\vec{b} = 5\hat{i}+2\hat{j}-14\hat{k}$ have the same initial point then their angular bisector having magnitude $\frac{7}{3}$ can be :

(1) $\frac{7}{3\sqrt{6}}(2\hat{i}+\hat{j}-\hat{k})$ (2) $\frac{7}{3\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (3) $\frac{7}{3\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$ (4) $\frac{7}{3\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$

23.2 One of the diagonals of a parallelopiped is $4\hat{j}-8\hat{k}$. If two diagonals of one of its base are $6\hat{i}+6\hat{k}$ and $4\hat{j}+2\hat{k}$, then its volume is

(1) 60 (2) 80 (3) 100 (4) 120

23.3 If $\vec{a}$ and $\vec{b}$ unequal unit vectors such that $(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})] = \vec{a}+\vec{b}$, then smaller angle θ between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) 0 (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

23.4 The distance between line $\vec{r} = (2\hat{i}-2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k}) = 5$ is

(1) $\frac{10}{9}$ (2) $\frac{10}{3\sqrt{3}}$ (3) $\frac{10}{3}$ (4) None of these

23.5 Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non zero and non coplaner vectors and $\vec{p},\vec{q}$ and $\vec{r}$ be three vectors given by $\vec{p} = \vec{a}+\vec{b}-2\vec{c}$, $\vec{q} = 3\vec{a}-2\vec{b}+\vec{c}$ and $\vec{r} = \vec{a}-4\vec{b}+2\vec{c}$. If the volume of parallelopiped determined by $\vec{a},\vec{b}$ and $\vec{c}$ is V_1 and volume of tetrahedron formed by $\vec{p},\vec{q}$ and $\vec{r}$ is V_2 then $V_2 : V_1$ =

(1) 5 : 2 (2) 2 : 5 (3) 5 : 3 (4) None of these

23.6 Vectors $4(\hat{i}+\hat{j}+\hat{k}), 7\hat{i}+6\hat{j}-\hat{k}$ and $3\hat{i}+2\hat{j}-5\hat{k}$ form

(1) Right angled triangle (2) Equilateral triangle
(3) Isosceles triangle (4) Scalene triangle

23.7 Number of integer values x for which vector $\vec{a} = x\hat{i}-3\hat{j}+\hat{k}$ and $\vec{b} = x\hat{i}-3x\hat{j}+2\hat{k}$ contain the obtuse angle between them.

(1) 1 (2) 0 (3) 3 (4) 4

23.8 $\vec{A},\vec{B},\vec{C}$ are unit vectors, suppose $\vec{A}.\vec{B} = \vec{A}.\vec{C} = 0$ and angle between $\vec{B}$ and $\vec{C}$ is $\frac{\pi}{6}$ then find k is $\vec{A} = k(\vec{B}\times\vec{C})$

(1) ± 2 (2) ±3 (3) ± $\frac{1}{3}$ (4) ± $\frac{1}{2}$

23.9 Let $\vec{A} = 2\hat{i}+\hat{k}, \vec{B}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}$ A vector $\vec{R}$ satisfying $\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$ would be

(1) $-\hat{i}-8\hat{j}+2\hat{k}$ (2) $\hat{i}+8\hat{j}-2\hat{k}$ (3) $2\hat{i}+16\hat{j}-4\hat{k}$ (4) $-2\hat{i}+16\hat{j}+4\hat{k}$

23.10 If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$, $\vec{c}\times\vec{a}=\vec{b}$ then

(1) $|\vec{a}|=|\vec{b}|=|\vec{c}|$ (2) $|\vec{a}|=2|\vec{b}|=|\vec{c}|$ (3) $|\vec{a}|\neq|\vec{b}|\neq|\vec{c}|$ (4) $|\vec{a}|=|\vec{b}|\neq|\vec{c}|$

23.11 Vector $\vec{a}=-4\hat{i}+3\hat{k}$

$\vec{b}=14\hat{i}+2\hat{j}-5\hat{k}$.

The vector $\vec{d}$ which is bisecting the angle between the vectors $\vec{a}$ and $\vec{b}$ and is having magnitude $\sqrt{6}$ is

(1) $\hat{i}+\hat{j}+2\hat{k}$ (2) $\hat{i}-\hat{j}+2\hat{k}$ (3) $\hat{i}+\hat{j}-2\hat{k}$ (4) none

23.12 If P is any point within a Δ ABC, then $\overrightarrow{PA}+\overrightarrow{CP}$ =

(1) $\overrightarrow{AC}+\overrightarrow{CB}$ (2) $\overrightarrow{BC}+\overrightarrow{BA}$ (3) $\overrightarrow{CB}+\overrightarrow{AB}$ (4) $\overrightarrow{CB}+\overrightarrow{BA}$

23.13 If ABCDEF is a regular hexagon and

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=\lambda\overrightarrow{AD}$ then λ =

(1) 2 (2) 3 (3) 4 (4) 6

23.14 If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\hat{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c} = \lambda\vec{a}+\mu\vec{b}$ then $\lambda + \mu$

(1) 0 (2) 1 (3) 2 (4) 3

23.15 If three unit vectors $\vec{a},\vec{b},\vec{c}$ are such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}}{2}$ then vector $\vec{a}$ makes angles with $\vec{b}$ & $\vec{c}$ respectively

(1) 60°, 90° (2) 45°, 45° (3) 30°, 60° (4) 90°, 60°

23.16 If $\vec{p}$ & $\vec{s}$ are not perpendicular to each other and $\vec{r}\times\vec{p}=\vec{q}\times\vec{p}$ & $\vec{r}.\vec{s}=0$ then $\vec{r}$ =

(1) $\vec{p}.\vec{s}$ (2) $\vec{q}-\left(\frac{\vec{q}.\vec{s}}{\vec{p}.\vec{s}}\right)\vec{p}$ (3) $\vec{q}+\left(\frac{\vec{q}.\vec{p}}{\vec{p}.\vec{s}}\right)\vec{p}$ (4) $\vec{q}+\mu\vec{p}$ for all scalars μ

23.17 If $\vec{a}, \vec{b}, \vec{c}$ are three non – coplanar & $\vec{p}, \vec{q}, \vec{r}$ are reciprocal vectors, then:

$$\left(\ell\vec{a}+m\vec{b}+n\vec{c}\right).\left(\ell\vec{p}+m\vec{q}+n\vec{r}\right)$$ is equal to :

(1) $\ell^2+m^2+n^2$ (2) $\ell m + m n + n \ell$ (3) 0 (4) none of these

23.18 If $\vec{a}$ is vector whose initial point divides the join of $5\hat{i}$ and $5\hat{j}$ in the ratio λ : 1 and terminal point is origin and $|\vec{a}|\leq\sqrt{17}$, then the set of exhaustive values of λ is

(1) $\left[-6,-\frac{1}{6}\right]$ (2) $\left(-\infty,\frac{1}{4}\right)\cup[4,\infty]$ (3) $\left[\frac{1}{4},4\right]$ (4) $\left[-\frac{1}{6},\infty\right]$

23.19 The vector equation of the plane containing the lines $\vec r = (\hat i + \hat j) + \lambda(\hat i + 2\hat j - \hat k)$ and $\vec r = (\hat i + \hat j) + \mu(-\hat i + \hat j - 2\hat k)$

(1) $\vec r.(\hat i + \hat j + \hat k) = 0$ (2) $\vec r.(\hat i - \hat j - \hat k) = 0$ (3) $\vec r.(\hat i + \hat j + \hat k) = 3$ (4) none

23.20 Angle between line $\vec r = (\hat i + 2\hat j - \hat k) + \lambda(\hat i - \hat j + \hat k)$ and the normal of plane $\vec r.(2\hat i - \hat j + \hat k) = 4$

(1) $\sin^{-1}\frac{2\sqrt2}{3}$ (2) $\cos^{-1}\left(\frac{2\sqrt2}{3}\right)$ (3) $\tan^{-1}\frac{2\sqrt2}{3}$ (4) $\cot^{-1}\frac{2\sqrt2}{3}$

23.21 The line of intersection of the planes $\vec r.(\hat i - 3\hat j + \hat k) = 1$ and $\vec r.(2\hat i + 5\hat j - 3\hat k) = 2$ is parallel to vector

(1) $-4\hat i + 5\hat j + 11\hat k$ (2) $4\hat i + 5\hat j + 11\hat k$ (3) $4\hat i - 5\hat j + 11\hat k$ (4) $4\hat i - 5\hat j - 11\hat k$

23.22 The value of $\frac{(\vec a \times \vec b)^2 + (\vec a.\vec b)^2}{2\vec a^2\vec b^2} =$

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{4}{3}$

23.23 If $\alpha(\vec a \times \vec b) + \beta(\vec b \times \vec c) + \gamma(\vec c \times \vec a) = \vec 0$ and atleast one of the number α, β and γ is non-zero, then vectors $\vec a, \vec b, \vec c$ are

(1) perpendicular (2) parallel (3) coplanar (4) none

Level : II (Tough)

23.24 If a,b,c are three non coplanar vector then

$\frac{\vec a.\vec b \times \vec c}{\vec c \times \vec a.\vec b} + \frac{\vec b.\vec a \times \vec c}{\vec c.\vec a \times \vec b} =$

(1) 0 (2) 2 (3) – 2 (4) 4

23.25 If the non-zero vectors $\vec a$ and $\vec b$ are perpendicular to each other, then solution of the equation $\vec r \times \vec a = \vec b$ is

(1) $\vec r = x\vec a + \frac{1}{\vec a.\vec a}(\vec a \times \vec b)$ (2) $\vec r = x\vec b - \frac{1}{\vec b.\vec b}(\vec a \times \vec b)$ (3) $\vec r = x\vec a \times \vec b$ (4) $\vec r = x\vec b \times \vec a$

23.26 Image of the point 'P' with position vector $7\hat i + \hat j + 2\hat k$ in the line whose vector equation is $\vec r = -3\hat j - 10\hat k + \lambda(4\hat i + 3\hat j + 5\hat k)$ has the position vector

(1) $-9\hat i + 5\hat j + 2\hat k$ (2) $9\hat i + 5\hat j - 2\hat k$ (3) $9\hat i - 5\hat j - 2\hat k$ (4) $9\hat i + 5\hat j + 2\hat k$

23.27 let $\vec a, \vec b, \vec c$ are three non-coplanar vectors such that

$\vec r_1 = \vec a - \vec b + \vec c$, $\vec r_2 = \vec b + \vec c - \vec a$, $\vec r_3 = \vec c + \vec a + \vec b$

$\vec r = 2\vec a - 3\vec b + 4\vec c$, If $\vec r = \lambda_1\vec r_1 + \lambda_2\vec r_2 + \lambda_3\vec r_3$ then $\lambda_1 + \lambda_2 + \lambda_3 =$

(1) 4 (2) 5 (3) 3 (4) 2

23.28 Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$

$\vec{b} = \hat{i} - \hat{j} - \hat{k}$

and $\vec{c}$ be a unit vector $\perp$ to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$ then $\vec{c}$ is

(1) $-\frac{1}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$ (2) $\frac{1}{\sqrt{6}}(\hat{i} - 2\hat{j} + \hat{k})$ (3) $\frac{1}{\sqrt{6}}(2\hat{i} - \hat{j} + \hat{k})$ (4) $-\frac{(2\hat{i} + \hat{j} + \hat{k})}{\sqrt{6}}$

23.29 A new tetrahedron is formed by joining the centroids of the faces of a given tetrahedron. The ratio of the volume of given tetrahedron to that of new tetrahedron is

(1) 3 (2) 9 (3) 27 (4) 81

22.30 A,B and C are three non collinear points with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively and plane ABC is not passing through origin, then vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are

(1) parallel vectors (2) non coplanar vector

(3) coplanar vector (4) linearly dependent vectors

SECTION - II : ASSERTION & REASONING TYPE

23.31 **Statement-1 :** If $\vec{a}$ is any vector in space then

$$\vec{a} = (\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$$

Statement-2 : $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$; $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

23.32 **Statement-1 :** If A,B,C,D are any four points in space then $|\overrightarrow{AB} \times \overrightarrow{CD} + \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}|$ is equal to 4Δ (Δ area of triangle ABC)

Statement-2 : Area of triangle formed by $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

23.33 **Statement-1 :** Vector $(-bc, b^2 + bc, c^2 + bc)$, $(a^2 + ac, -ac, c^2 + ac)$ and $(a^2 + ab, b^2 + ab, -ab)$ are coplanar where a,b,c are non-zero then $ab + bc + ca = 0$

Statement-2 : $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $[\vec{a}\vec{b}\vec{c}] = 0$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

TOPIC 24 THREE DIMENSIONAL GEOMETRY

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

24.1 If the incident ray and normal have the directions of the vectors (1, –3, 1) , (1,1,1) respectively, then direction of the reflected ray is -

(1) (4, –8 , 4) (2) (5, –7 , 5) (3) (6, –6 , 6) (4) (–5 , 7, 5)

24.2 The plane lx+my = 0 is rotated about its line of intersection with XOY plane through an angle 'α', If the equation of the plane is $\ell x + my + nz = 0$, then n is equal to -

(1) $\pm\sqrt{(\ell^2+m^2)}\cos\alpha$ (2) $\pm\sqrt{(\ell^2+m^2)}\sin\alpha$ (3) $\pm\sqrt{(\ell^2+m^2)}\tan\alpha$ (4) None of these

24.3 System of equation $x + 3y + 2z = 6$
$x + \lambda y + 2z = 7$
$x + 3y + 2z = \mu$ has

(1) unique solution if $\lambda = 2, \mu \neq 6$ (2) no solution if $\lambda = 4, \mu = 6$
(3) infinite if $\lambda = 5, \mu = 7$ (4) no solution if $\lambda = 3, \mu = 5$

24.4 Points (–2, 4, 7), (3, – 6, – 8) and (1, –2, –2) are

(1) collinear (2) vertices of an equilateral triangle
(3) vertices of isosceles triangle (4) none of these

24.5 If projection of a line on x, y and z axis are 6, 2 and 3 respectively, then d.c.s. of line is

(1) $\left(\frac{6}{7},\frac{2}{7},\frac{3}{7}\right)$ (2) $\left(\frac{3}{5},\frac{5}{7},\frac{6}{7}\right)$ (3) $\left(\frac{1}{7},\frac{2}{7},\frac{3}{7}\right)$ (4) none of these

24.6 Direction ratios of two lines are a,b,c and $\frac{1}{bc},\frac{1}{ca},\frac{1}{ab}$ then lines are

(1) perpendicular (2) parallel (3) coincident (4) none

24.7 The equation of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ will be

(1) $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$ (2) $\frac{x-1}{-2}=\frac{y-2}{3}=\frac{z+4}{8}$

(3) $\frac{x-1}{3}=\frac{y-2}{2}=\frac{z+4}{8}$ (4) none

24.8 The point of intersection of lines

$\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{1}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is

(1) (–1, –1, –1) (2) (–1, –1, 1) (3) (1, –1, –1) (4) (–1, 1, –1)

24.9 Shortest distance between lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

(1) $\sqrt{30}$ (2) $2\sqrt{30}$ (3) $5\sqrt{30}$ (4) $3\sqrt{30}$

24.10 Equation of plane which is parallel to y axis and cuts off intercepts of length 2 and 3 from x axis and z axis is

(1) $3x + 2z = 1$ (2) $3x + 2z = 6$ (3) $2x + 3z = 6$ (4) $3x + 2z = 0$

24.11 The equation of the plane which meets the coordinate axes in A, B, C such that the centroid of the ΔABC is the point (p, q, r) is

(1) $\frac{x-p}{p}$ (2) $\frac{x-q}{q}=\frac{x-r}{r}$ (3) $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$ (4) $px + qy + zr = 1$

24.12 The direction ratios of a normal to the plane passing through (1, 0, 0), (0, 1, 0) and making an angle $\frac{\pi}{4}$ with the plane $x + y = 3$ are

(1) $1, \sqrt{2}, 1$ (2) $1, 1, \sqrt{2}$ (3) 1, 1, 2 (4) $\sqrt{2}, 1, 1$

24.13 Equation, $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$ represents a sphere, if:

(1) $a = b = c$ (2) $f = g = h = 0$
(3) $v = u = w$ (4) $a = b = c$ & $f = g = h = 0$

24.14 The equation of the plane through the point (–1, 2 , 0) and parallel to the lines $\frac{x}{3}=\frac{y+1}{0}=\frac{z-2}{-1}$ and $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$ is

(1) $x + 2y + 3z - 3 = 0$ (2) $x - 2y + 3z + 5 = 0$
(3) $x + 2y + 3z - 1 = 0$ (4) $x + y + 3z - 1 = 0$

24.15 If the planes $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ pass through a line, then the value of $a^2 + b^2 + c^2 + 2abc$ is

(1) 0 (2) 1 (3) 2 (4) 8

24.16 The centre of the circle $|\vec{r}-\hat{i}+2\hat{j}+\hat{k}| = 5$ and $\vec{r}.(2i+2j-k) = 8$ is:

(1) $3\hat{i}-2\hat{k}$ (2) $\hat{i}+2\hat{j}-2\hat{k}$ (3) $4\hat{i}$ (4) none of these

24.17 If co-ordinates of points A,B,C,D be (2, 3, –1) (3, 5, –3) (1, 2, 3) and (3, 5, 7) respectively then projection of AB on CD is

(1) $\sqrt{3}$ (2) $\sqrt{3}-\frac{3}{\sqrt{3}}$ (3) $\frac{9\sqrt{3}}{3}$ (4) $3\sqrt{3}$

24.18 Which of the following planes intersects the planes $x - y + 2z = 3$ and $4x + 3y - z = 1$ along the same line?

(1) $11x + 10y - 5z = 0$ (2) $7x + 7y - 4z = 0$
(3) $5x + 2y + z = 2$ (4) none of these

24.19 The sum of coordinates of a point lying in yz-plane is 3. If its distance from xz-plane is twice its distance from xy-plane, then its coordinates are

(1) (0, 1, 2) (2) (0, 2, 1) (3) (0, –1, –2) (4) (0, 5, –3)

24.20 The angle between two body diagonals of a cube is

(1) $\cos^{-1}\left(\frac{4}{5}\right)$ (2) $\cos^{-1}\left(\frac{2}{3}\right)$ (3) $\sin^{-1}\left(\frac{1}{3}\right)$ (4) $\cos^{-1}\left(\frac{1}{3}\right)$

24.21 If ΔABC is a right angles iscosceles triangle which is right angled at B(–1, 6, 6). If A is (a, 7, 10) and circumcenter of ΔABC is (–2, 8, 8) then 'a' is :

(1) 0 (2) 1 (3) 2 (4) 3

Level : II (Tough)

24.22 The equation of the plane through the line of intersection of planes ax + by + cz + d = 0, a′x + b′y + c′z + d′ = 0 and parallel to the line y = 0, z = 0 is

(1) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd) = 0$ (2) $(ab' - a'b)x + (bc' - b'c)y + (ad' - a'd)z = 0$

(3) $(ab' - a'b)y + (ac' - a'c)z + (ad' - a'd) = 0$ (4) $(ab' + a'b)y + (ac' + a'c)z + (ad' + a'd) = 0$

24.23 The equation of the bisector of the acute angle between planes 3x – 6y + 2z + 5 = 0 and 4x – 12y + 3z – 3 = 0 is

(1) 11x + 6y – 5z + 86 = 0 (2) 33x – 13y + 32z + 45 = 0

(3) 67x – 162y + 47z + 44 = 0 (4) 33x + 13y + 32z + 45 = 0

24.24 The image of the point $P(\alpha, \beta, \gamma)$ by the plane $\ell x + my + nz = 0$ is $Q(\alpha', \beta', \gamma')$ then

(1) $\alpha^2 + \beta^2 + \gamma^2 = \ell^2 + m^2 + n^2$ (2) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha')^2 + (\beta')^2 + (\gamma')^2$

(3) $\alpha\alpha' + \beta\beta' + \gamma\gamma' = 0$ (4) $\ell(\alpha - \alpha') + m(\beta - \beta') + n(\gamma - \gamma') = 0$

24.25 Equation of a line passing through the point (–3, 2, – 4) and equally inclined to the axes are

(1) x + 3 = y + 2 = z– 4 (2) x + 3 = y – 2 = z + 5

(3) $\frac{x+3}{-2} = \frac{y-2}{-2} = \frac{z+4}{-2}$ (4) $\frac{x+3}{-2} = \frac{y+2}{-2} = \frac{z-4}{-2}$

24.26 The equation of a plane is 2x – y – 3z = 5 and A(1, 1, 1), B(2, 1, –3), C(1, –2, –2) and D(–3, 1, 2) are four points. Which of the following line segments are intersected by the plane?

(1) AD (2) AB (3) BC (4) none of these

24.27 The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, –7) is

(1) x + 2y – z = 0 (2) x + y + z = 0 (3) 3x + y + z = 0 (4) 4x + y + z = 0

24.28 lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are

(1) coincident (2) Perpendicular

(3) Intersect only at one point (4) Non intersecting

24.29 The plane 2x + 5y – 4z – 6 = 0 and the line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+2}{3}$, then equation of the image of the given line with respect to the given plane is

(1) $\frac{x+2}{79} = \frac{y+2}{40} = \frac{z+5}{-247}$ (2) $\frac{x+2}{79} = \frac{y+2}{40} = \frac{z+5}{247}$

(3) $\frac{x-1}{2} = \frac{y-2}{5} = \frac{z+2}{-4}$ (4) $\frac{x+2}{2} = \frac{y+2}{5} = \frac{z+5}{4}$

SECTION - II : ASSERTION & REASONING TYPE

24.30 **Statement - 1 :** The lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and $\frac{9x-16}{13} = \frac{9y-1}{7} = \frac{z}{-1}$ are coplanar

Statement - 2 : Two lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar

if $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ a_2-a_1 & b_2-b_1 & c_2-c_1 \end{vmatrix} = 0$

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

24.31 Consider the points A (0, 0,0), B(2, 0, 0), C(1, $\sqrt{3}$, 0) and D $\left(1, \frac{1}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right)$

Statement-1 : ABCD is a square.
Statement-2 : |AB| = |BC| = |CD| = |DA|.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

24.32 Consider the lines

$\ell_1 : \frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$ and $\ell_2 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-1}$

Statement-1 : Shortest distance between the lines ℓ_1 and ℓ_2 is 0.
Statement-2 : The lines ℓ_1 and ℓ_2 intersect in the point (1, 2,3).
which of the following is correct
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

24.33 Consider the sphere

S : $x^2 + y^2 + z^2 = 25$ and the plane p : $x + y + z = 4\sqrt{3}$

Statement-1 : The sphere 'S' and the plane 'p' have no point in common.
Statement-2 : The distance of the plane p from centre of S is less than the radius of S.
which of the following is correct
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

TOPIC 25 TRIGONOMETRIC IDENTITIES & EQUATION

SECTION - I : STRAIGHT OBJECTIVE TYPE

Level : I (Easy/Moderate)

25.1 The number of solution of the equation tanx + sec x = 2cosx lying in the interval $[0, 2\pi]$, is
(1) 3 (2) 2 (3) 1 (4) 0

25.2 Let a, b, c be the sides of a triangle whose perimeter is P and area is A, then
(1) $P^3 \le 27\,(b+c-a)\,(c+a-b)\,(a+b-c)$ (2) $P^2 \le 3(a^2+b^2+c^2)$
(3) $a^2 + b^2 + c^2 \ge 8\sqrt{3}\,A$ (4) $P^4 \le 256\,A$

25.3 The value of the expression $\dfrac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$ is
(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) none of these

25.4 If $\tan A = \dfrac{1-\cos B}{\sin B}$ then
(1) tan 3A = tan B (2) tan 2A = tan B (3) tan 3A = tan2B (4) None of these

25.5 If $\frac{\pi}{2} < x < \pi$ then the value of the expression $\sqrt{\dfrac{1-\sin x}{1+\sin x}} + \sqrt{\dfrac{1+\sin x}{1-\sin x}}$ is
(1) $\dfrac{2}{\cos x}$ (2) $\dfrac{1}{\sin x}$ (3) $-\dfrac{2}{\cos x}$ (4) non existing

25.6 If $x = \sin^8\theta + \cos^{14}\theta$ then
(1) $x \ge 1$ (2) $0 \le x \le 1$ (3) $0 < x \le 1$ (4) none of these

25.7 If $\tan\theta = \sqrt{\dfrac{3}{2}}$, then the sum of the series $1 + 2(1-\cos\theta) + 3(1-\cos\theta)^2 + 4(1-\cos\theta)^3 + \ldots\ldots$ is
(1) $\frac{2}{3}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{5}{2\sqrt{2}}$ (4) $\frac{5}{2}$

25.8 In a ΔABC if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$ where a,b,c are lengths of sides of ΔABC, then value of $\sin^2 A + \sin^2 B + \sin^2 C$ is
(1) $\frac{4}{9}$ (2) $\frac{9}{4}$ (3) $3\sqrt{3}$ (4) 1

25.9 tan 20° + 2 tan 50° is equal to
(1) tan 70° (2) cot 70° (3) sin 70° (4) tan 30°

25.10 Least value of $\tan^4\phi + \cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + \cot^4\phi + 2$ is

(1) 4 (2) 6 (3) $6 - \sqrt{10}$ (4) $6 + \sqrt{10}$

25.11 The value of $\cos\frac{6\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{\pi}{7}$ is

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) 0 (4) 1

25.12 The value of $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n}$ + ... upto n terms is -

(1) 0 (2) 1 (3) $\frac{1}{2}$ (4) none of these

25.13 The equation $2\cos\phi - 3\sin\phi = k$ in ϕ has real solution then -

(1) $k = -4$ (2) $|k| < \sqrt{13}$ (3) $|k| < 5$ (4) $|k| = 5$

25.14 Number of values of $x \in R$ which satisfy the equation $\cos(\pi\sqrt{x-4})\cos(\pi\sqrt{x}) = 1$ is -

(1) 1 (2) 0 (3) 2 (4) none of these

25.15 If $x \in \left(0, \frac{\pi}{2}\right]$ then $\sin 5x + \sin 3x + \sin x = 0$ is true for

(1) $x = \frac{\pi}{6}$ (2) $x = \frac{\pi}{12}$ (3) $x = \frac{\pi}{3}$ (4) $x = \frac{\pi}{9}$

25.16 The solution of inequality $\cos 2x \le -\sin x$ is -

(1) $x \in \left\{(2n+1)\frac{\pi}{2}\right\}; n \in I$

(2) $x \in \left\{(4n+1)\frac{\pi}{2}\right\} \cup \left[2n\pi - \frac{\pi}{2}, 2n\pi - \frac{\pi}{6}\right]; n \in I$

(3) $x \in \left[2n\pi - \frac{\pi}{2}, 2n\pi - \frac{\pi}{6}\right]; n \in I$

(4) $x \in \left(2n\pi - \frac{\pi}{2}, 2n\pi - \frac{\pi}{6}\right) \cup \left\{(4n+1)\frac{\pi}{2}\right\}; n \in I$

25.17 General solution of $\sin x + \sin 5x = \sin 2x + \sin 4x$ is

(1) $\frac{n\pi}{3}; n \in I$ (2) $2n\pi, \frac{2n\pi}{3}; n \in I$ (3) $2n\pi; n \in I$ (4) none of these

25.18 $\sin 3\theta = 4\sin\theta\sin 2\theta\sin 4\theta$ in $0 \le \theta \le \pi$ has

(1) 2 real solutions (2) 4 real solutions (3) 6 real solutions (4) 8 real solutions

25.19 The value of $\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cos\frac{4\pi}{10}\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$ is -

(1) $\frac{\sqrt{10+2\sqrt{5}}}{64}$ (2) $\frac{\cos\frac{\pi}{10}}{16}$ (3) $\frac{\cos\frac{\pi}{10}}{16}$ (4) $-\frac{\sqrt{10+2\sqrt{5}}}{64}$

25.20 If $0 < \theta < \frac{\pi}{2}$ then $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4\theta}}}$ is equal to -

(1) $2\sin\frac{\theta}{2}$ (2) $2\cos 2\theta$ (3) $2\sin\theta$ (4) $\sqrt{2}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)$

25.21 The value of

(1) $\frac{-1}{4(\sqrt{5}-1)}$

25.22 Value of

(1) 1

25.23 If 1 + sin x +

(1) $\frac{\pi}{6}$

Level : II (Tough

25.24 A regular h

is $\sqrt{3}-1$,

(1) $\sqrt{2}+1$

25.25 In a right a

other angl

(1) $\frac{\pi}{3}, \frac{\pi}{6}$

25.26 The value

(1) cot 3

25.27 If cos α

(1) a^2 +

25.28 $\frac{1}{\cos 29}$

(1) $\frac{2\sqrt{}}{3}$

25.29 If three

cos A

(1) $\frac{1}{12}$